ICEBERG SEMANTICS

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Iceberg semantics is meant to be a useful framework for studying and developing theories of mass-count, singularity-plurality for lexical nouns, complex nouns (NPs) and noun phrases (DPs).
Main inspirations:
-Link 1984, 1984 and other's mereological approach to singularity and plurality
-Chierchia 1998's approach to mass nouns (but not Chierchia 2010)
-the overlap based approaches to mass and count nouns by Rothstein 2010, 2011, Landman 2011
-Some aspects of Krifka 1989's approach to countability.
-Persistent questions by Barbara Partee about atoms *versus* minimal elements.

0. Background notions

-**Background model:** a complete atomic Boolean algebra **B** = <B, $\sqsubseteq$, $\neg$, $\sqcup$, $\sqcap$, 0, 1>, with set of atoms, ATOM, and operations of complete join and meet, $\sqcup$ and $\sqcap$.

-**Boolean part set:** $(x] = \{b \in B: b \sqsubseteq x\}$ the set of all parts of x.
-Atomic part set: $\mathbf{atom}(x) = (x] \cap \text{ATOM}$ the set of all atomic parts of x.

[Obvious generalizations: $(Z] = \cup\{(x]: x \in Z\}$; $\mathbf{atom}(Z) = (Z] \cap \text{ATOM}$]

-**Closure under sum (Link 1983):**
$^{*}Z = \{b \in B: \exists Y \subseteq Z: b = \sqcup Y\}$
the set of all sums that can be formed with the elements of Z.

Definite operator (Sharvy 1980):

$$\sigma(Z) = \begin{cases} \sqcup Z & \text{if } \sqcup Z \in Z \\ \perp & \text{otherwise} \end{cases}$$

1. Mountains.

Link 1983: **Atomicity** as the central notion in theory of count nouns.

Singular and plural nouns

cat	$\rightarrow$	CAT where CAT $\subseteq$ ATOM	Singular nouns denote sets of atoms
cats	$\rightarrow$	*CAT	Semantic pluralization is closure under $\sqcup$

Cardinality as counting of atomic parts:

$|x| = |$**atom**$(x)|$ — The number of **atomic** parts of x.

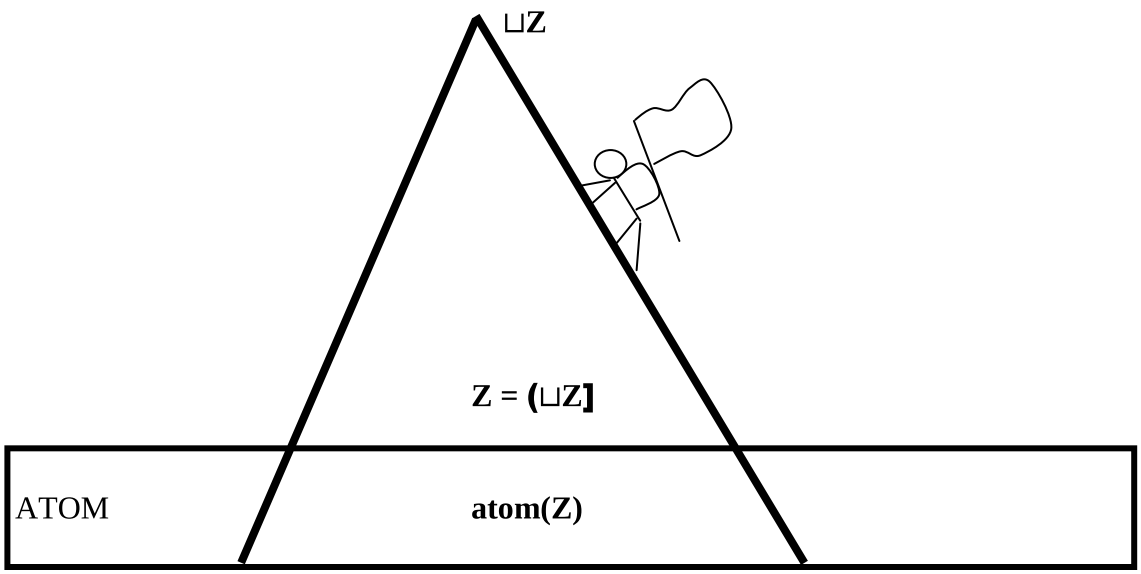
Restriction to atomic predicates as a felicity constraint: e.g. determiner *each***:**

$$each \rightarrow \lambda Q \lambda P. \begin{cases} Q \subseteq P & \text{if } Q \subseteq \textbf{ATOM} \quad \text{the input is a set of atoms} \\ \text{undefined} & \text{otherwise} \end{cases}$$

Distributive operator: distribution = distribution to **atoms**:

$\lambda P \lambda x. \forall a \in$ **atom**(x): $P(a)$

Mountain semantics: the semantics of nouns and noun phrases is grounded in sets of atoms



Counting: you count plurality x in the denotation of count noun *cats* by counting the atomic parts of x

2. Icebergs.

Alternative view: ***cats-minimal elements*** instead of ***atomic cats***:

Pluralities of cats are built from singular cats.
Singular cats are **minimal** within the set of cats, their parts do not count as cats .

Provisional definitions: Let $x \in Z - \{0|$

x is Z-**minimal** iff for all $y \in Z-\{0\}$: if $y \sqsubseteq x$ then $y=x$ (x has no proper parts in Z)
min(Z) is the set of Z-minimal elements of Z.

$\mathbf{min}_Z(x) = (x] \cap \mathbf{min}(Z)$ The parts of x that are minimal in Z
$\mathbf{card}_Z(x) = |\mathbf{min}_Z(x)|$ Cardinality of x = the number of z-minimal parts of x

A plurality x of cats counts as *six cats* iff
there are 6 parts of x that are minimal in the set of cats.

In mountain semantics, these two notions are made to coincide:
Mountain semantics: - Singular predicates are sets of atoms,
-plural predicates are the closure under sum of singular predicates
Hence: If $x \in$ *CAT then $\mathbf{min}_{*CAT}(x) = \mathbf{atom}(x)$

(Not every semantics for count nouns following Link is Mountain semantics: e.g. Krifka 1989)

What happens to the semantics of count nouns if we loosen the notion of Z-minimal part from the notion of atom?

For counting we must preserve from the notion of atoms the requirement that the set of minimal elements is a *disjoint* set.

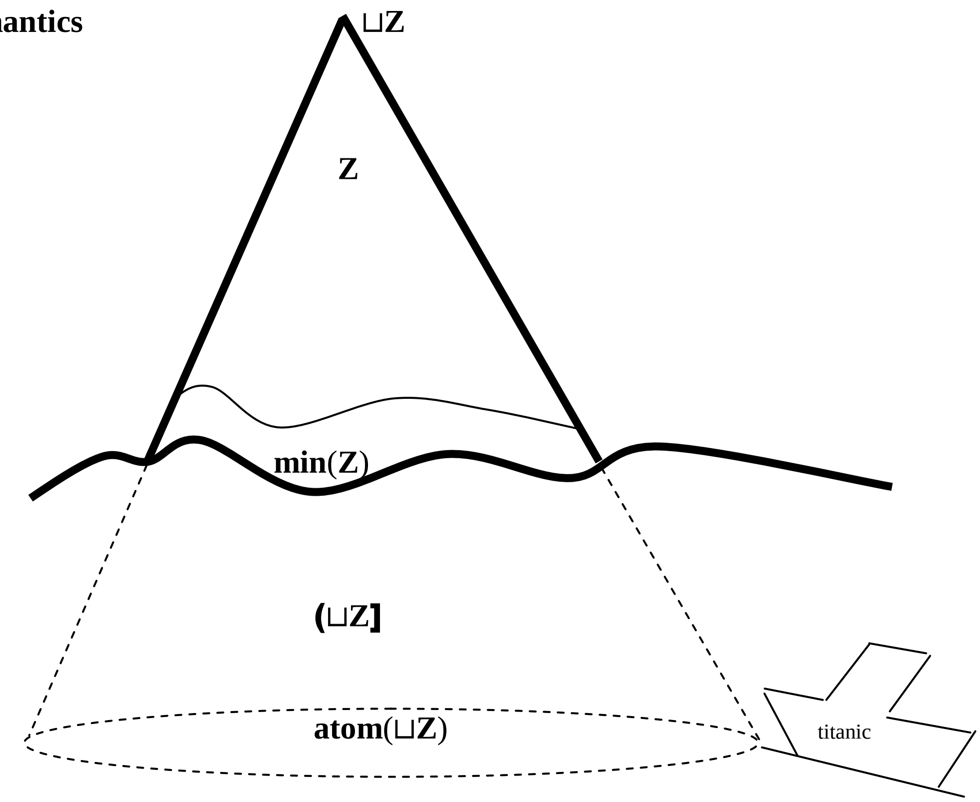
a and b **overlap** iff $a \sqcap b \neq 0$ $(a,b \in B - \{0\})$
a and b are **disjoint** iff a and b do not overlap

Z **overlaps** iff for some $a,b \in Z$: a and b overlap $(Z \subseteq B - \{0\})$
Z is **disjoint** iff Z does not overlap

Disjoint semantics for count nouns:
cat $\rightarrow$ CAT where CAT is a **disjoint** subset of B
cats $\rightarrow$ *CAT

The denotation is still a mountain, but no longer a mountain solidly attached to the bottom of the structure. This means that it floats. Like an iceberg:

Iceberg Semantics



Problem: Distributivity.
Mountain semantics: Distributivity distributes to atoms.
Iceberg semantics: Distributivity distributes to minimal elements.

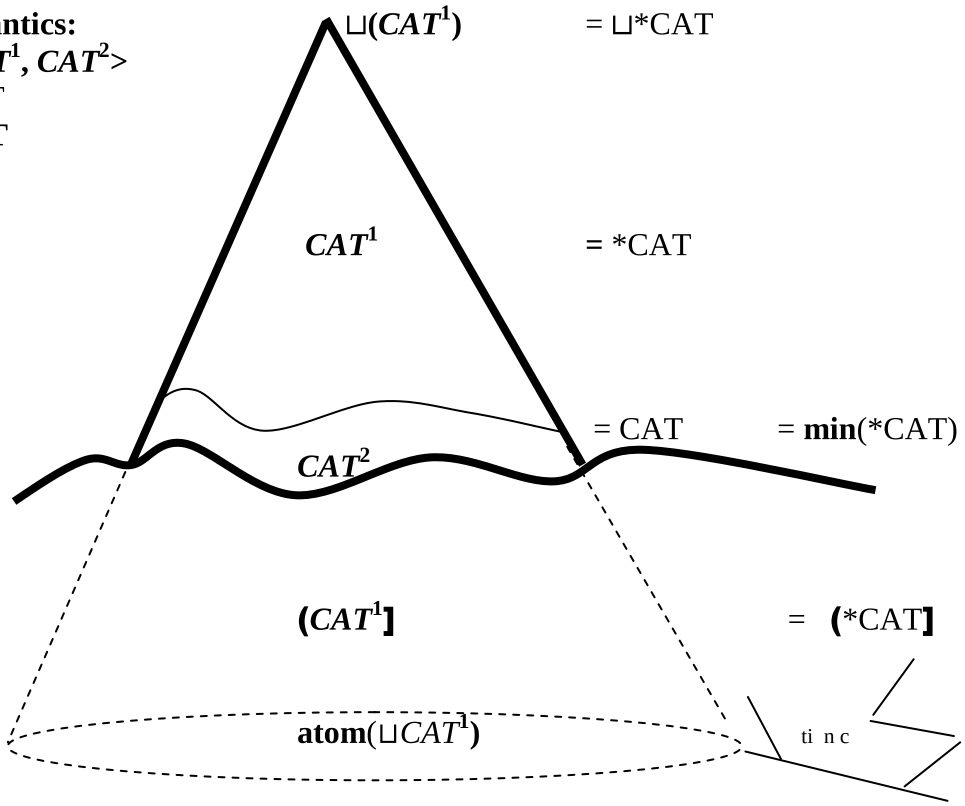
Problem: the DP interpretation is an object in B.
The information about which noun interpretation was used to derive the object is lost.
Hence also the set of minimal elements.

e.g. ***Each*** *of the cats* must distribute to CAT, the set of minimal elements in *CAT.
The interpretation of *the cats*, σ(*CAT), is the sum of many sets, ⊔CAT, ⊔**atom**(⊔CAT))….
The semantics does not know **what set** to distribute to.

Conclusion: in Iceberg semantics of count nouns, we must *keep track* of what we distribute to.

Iceberg Proposal: Interpret nouns as **pairs** $N = <N^1, N^2>$:
N^1 – the first-tier interpretation – is the familiar set interpretation
N^2 – the second-tier interpretation – is the set relative to which the elements of the first tier interpretation are counted, and relative to which distribution takes place.

Iceberg Semantics:
CAT **=** $<\boldsymbol{CAT}^1, \boldsymbol{CAT}^2>$
$\boldsymbol{CAT}^1$ = *CAT
$\boldsymbol{CAT}^2$ = **CAT**



Idea in a nutshell:

cats $\rightarrow$ <*CAT, CAT>, with CAT a disjoint set.

the cats $\rightarrow$ <σ(*CATS), CAT>

Distribution is to the parts of σ(*CAT) (first tier) that are in CAT (second tier).

3. Iceberg semantics for mass nouns and count nouns

X **generates** Z **under** $\sqcup$ iff $Z \subseteq {}^*X$ (X, Z $\subseteq$ B)
Z **is grounded** X iff X generates Z under $\sqcup$ and $X \subseteq Z$.

Iceberg semantics for *lexical* nouns: *N is a grounded set*:
lexical noun *NOUN* $\rightarrow$ $N = \langle N^1, N^2 \rangle$, where N^1 is a set grounded in set N^2

[Fact: If Z is grounded in X then **min**(Z) $\subseteq$ X (because generation is under $\sqcup$).]

Iceberg semantics for complex nouns: *NP* is a generated set:
noun phrase NP $\rightarrow$ $NP = \langle NP^1, NP^2 \rangle$, where NP^1 is a set generated by set NP^2

Complex noun interpretations are not necessarily grounded:
e.g. *three cats* $\rightarrow$ $\langle \lambda x.{}^*CAT(x) \wedge |x|=3, CAT \rangle$
$\lambda x.{}^*CAT(x) \wedge |x|=3$ is generated by CAT, but not grounded in it,
because CAT is not a subset of $\lambda x.{}^*CAT(x) \wedge |x|=3$.

Let *X* be a **generated** set.
X is **count** iff X^2 is **disjoint**
X is **mass** otherwise

Let *X* be **count**.
X is **singular** iff $X^1 = X^2$
X is **plural** otherwise

Let *X* be **mass**.
X is **neat** iff **min**(X^2) is **disjoint**
X is **mess** otherwise

Landman 2011: Prototypical mass nouns like *water, wall paper* and *mud* are **mess mass nouns**.
They are built from overlapping minimal generators. (More on this in appendix 3)

Mass nouns like *furniture, kitchenware* are **neat mass nouns**.
Count: *Ronya* counts as "one item" of *cat*, and so does *Minoes*.
Ronya and Minoes does not count as "one item" of *cat*
Neat mass: *The cup* counts as "one item" of *kitchenware*, and so does *the saucer*.
The cup and saucer **also** counts as "one item" of *kitchenware*.

Central idea: Count nouns and neat mass nouns are generated by the elements that count as "one item".
Hence: the generator set of count nouns like *cat* is disjoint.
the generator set of neat mass nouns like *kitchenware* is not disjoint.

Neat mass nouns *versus* mess mass nouns: more in Landman 2011 and appendices below.
Until then I will, for ease, pretend that all mass nouns are neat.

Lexical singular count nouns:
cat → <CAT, CAT> where CAT is disjoint.

Lexical plural count nouns:
cats → <*CAT, CAT> (*CAT is grounded in CAT)

Lexical (neat) mass nouns:
Nice triple in Dutch:

meubel	singular count noun	*item of furniture*
meubels	plural count noun	*items of furniture*
meubilair	mass noun	*furniture*

Chierchia 1998:

meubel	MEUBEL	where MEUBEL $\subseteq$ ATOM
meubels	*MEUBEL − MEUBEL	
meubilair	*MEUBEL	

Much discussed in the literature: unsatisfactory semantics of plural nouns.
This aspect of Chierchia's proposal is easily avoided in Iceberg semantics:

-singular and plural count nouns share the second tier interpretation:

meubel → < MEUBEL, **MEUBEL**> with MEUBEL **disjoint** (in context).
meubels → <*MEUBEL, **MEUBEL**>

-neat mass nouns and plural nouns share the first tier interpretation:
meubels → <***MEUBEL**, MEUBEL>
meubilair → <***MEUBEL**, *MEUBEL>

-the essential similarity between neat mass nouns and singular nouns is expressed by identity of 'singular' form:

< α , α >
meubel → < MEUBEL, MEUBEL>
meubilair → <*MEUBEL, *MEUBEL>

Both have singular form, but the first is count, while the second is not, since *MEUBEL is not disjoint.

[Caveat: I am, of course, assuming (and suppressing for simplicity) here a properly **modal** formulation of all these notions, nouns are not count in some worlds and mass in other worlds.]

Presuppositional cardinality: we count y $\in X^1$ in terms of the cardinality of the set of its X^2-parts, ***on the condition that this set is disjoint.***

> Let X be a generated set and y $\in X^1$:
> $(y] \cap X^2$ is **the set of X^2-parts of** y

With this we define the ***X*-count set of y**, the set of parts of y in terms of which y is counted as X:

> **count**$_X$(y) is the X-count set of y:
>
> $$\mathbf{count}_X(y) = \begin{cases} (y] \cap X^2 & \text{if } X^2 \text{ is } \mathbf{disjoint} \\ \perp & \text{otherwise} \end{cases}$$

The X-count set of y is the set of X^2-parts of y **if X^2 is disjoint**.
Hence the notion **count**$_X$(y) is only defined for generated sets that are **count**.

The notion **count**$_X$(y) is used in the definition of the cardinality function.
As a consequence, the cardinality function **requires a count** input.

> **Cardinality function:**
>
> $$\mathbf{card} = \lambda X\, \lambda y. \begin{cases} |\mathbf{count}_X(y)| & \text{if } X^2 \text{ is disjoint and } y \in X^1 \\ \perp & \text{otherwise} \end{cases}$$
>
> **card** maps X onto the function that maps every y in X^1 onto the number of its X^2-parts

Example 1.

cats $\rightarrow$ *CAT* = <*CAT, CAT>, with CAT a disjoint set.
Ronya, Minoes $\in$ CAT

Since CAT is disjoint, **count**$_{CAT}$(Ronya $\sqcup$ Minoes) is defined.

count$_{CAT}$(Ronya $\sqcup$ Minoes) = (Ronya $\sqcup$ Minoes] $\cap$ CAT = {Ronya, Minoes}.
card$_{CAT}$(Ronya $\sqcup$ Minoes) = |**count**$_{CAT}$(Ronya $\sqcup$ Minoes)| = |{Ronya, Minoes}| = 2

Example 2.

furniture $\rightarrow$ *MEUBEL* = <*MEUBEL,*MEUBEL>, with *MEUBEL not disjoint.
The Dresser, The Pianola $\in$ MEUBEL (i.e. they are pieces of furniture)

count$_{MEUBEL}$(The Dresser $\sqcup$ The Pianola) is undefined.
card$_{MEUBEL}$ (The Dresser $\sqcup$ The Pianola) is undefined.

4. Iceberg Semantics for DPs

An **object** is a pair $x = \langle x^1, x^2 \rangle$ with x^1 an element of B and x^2 a subset of B
A **regular object** is an object x where $x^1 = \sqcup x^2$ (the element is the sum of the set)

A **count object** is a regular object x where $\boldsymbol{x^2}$ **is disjoint**.
A **mass object** is a regular object that is **not count**.

A **singular object** is a count object x where $\boldsymbol{x^2 = \{x^1\}}$.
A **plural object** is a count object that is **not singular**.

Semantics:

Proper names:
Ronya $\rightarrow$ <Ronya, {Ronya}> with Ronya $\in$ B.

We interpret *Ronya* as a **singular count object**.

***Sum conjunction*:**

$and \rightarrow \lambda x \lambda y.\ \langle x^1 \sqcup y^1, x^2 \cup y^2 \rangle$

Hence:
Ronya and Minoes $\rightarrow$ <Ronya $\sqcup$ Minoes, {Ronya, Minoes}>
We interpret *Ronya and Minoes* as a plural count object.

***The definite article*:**

$the \rightarrow \quad \lambda x.\ \langle \sigma(x^1), x^2 \rangle$

Hence indeed:
the cat $\rightarrow$ <σ(CAT), CAT>
σ(CAT) is undefined, unless CAT = {σ(CAT)}, so:

the cat $\rightarrow$ <σ(CAT), {σ(CAT)}> *The cat* denotes a **singular count object**.

the cats $\rightarrow$ <σ(*CAT),CAT> *The cats* denotes a **plural count object**.

the furniture $\rightarrow$ <σ(*MEUBEL),*MEUBEL> *The furniture* denotes a **mass object**.

The generalized quantifier each

$$each \to \lambda X. \begin{cases} <\lambda P.\ X^1 \subseteq P,\ X^2> & \text{if } X \text{ is } \mathbf{singular} \\ \text{undefined} & \text{otherwise} \end{cases}$$

The input of *each* is required to be semantically singular. This can be an individual level singular predicate like *cat*, but also a group level singular predicate like *committee*.
Infelicitous are:

#*Each cats* because <*CAT, CAT> is plural
#*Each furniture* because <*MEUBEL,*MEUBEL> is mass.

Obvious definitions of $\mathbf{count}_x(x)$ and $\mathbf{card}_x(x)$ for regular objects.
Here we prefer the notation $\mathbf{count}_{x2}(x)$ and $\mathbf{card}_{x2}(x)$.

5. The operation of singular shift ↑

Group formation, as used by Link 1984, Landman 2000, and many others maps sums onto group **atoms**. Two kinds of uses:

Singularization: Landman 2000:
-group readings of plural noun phrases are semantically singular readings
-distribution to subgroups (semantic grid)

Externalization: Landman 2007: Dual-perspective intensionality:
-entities ordered extensionally **inside** a part-of structure *versus*
entities-in-intension, lifted out of their part-of structure, treated as more than the sum of their parts.

Iceberg semantics: these operations can be separated into two:
singular shift (this section) and externalization (appendix 1).
Singular shift: a shifting operation that applies to DP interpretations making them **semantically singular**:

Singular shift: ↑ is an operation from regular objects to regular objects:
$\uparrow(x) = <x^1, \{x^1\}>$ (improved version below)

Singular shift keeps the x^1 part, but singularizes it (on the second tier).

Fact: ↑ is *identity* on objects that are already singular:

$$\uparrow(<\text{Ronya},\ \{\text{Ronya}\}>) = <\text{Ronya},\ \{\text{Ronya}\}>$$

Shifting ambiguity for plural DPs:

the cats $\rightarrow$ $<\sigma(*CAT), CAT>$ **Plural object**
$\rightarrow \uparrow(<\sigma(*CAT), CAT>)$
$= <\sigma(*CAT), \{\sigma(*CAT)\}>$ **Singular collective object**

$\mathbf{card}_{\{\sigma(*CAT)\}}(<\sigma(*CAT), \mathbf{\{\sigma(*CAT)\}}>) = |\{\sigma(*CAT)\}| = 1$

Mass objects pattern with plural objects:

the furniture $\rightarrow$ $<\sigma(*MEUBEL), *MEUBEL>$ **Mass object**
$\rightarrow \uparrow(<\sigma(*MEUBEL), *MEUBEL>)$
$= <\sigma(*MEUBEL), \{\sigma(*MEUBEL)\}>$ **Singular collective count object**

The existence of a collective count reading was argued in Landman 1991. Cf:

(1) *Both* the coffee in the cup and the coffee in the pot had strychnine in them.

Both = ***each*** on a domain of two. It requires a sum of ***count*** objects as input.

Sketch of analysis:
C_{CUP}: generating set for the coffee in the cup
C_{POT}: generating set for the coffee in the pot

the coffee in the cup $\rightarrow$ $<\sigma(*C_{CUP}), *C_{CUP}>$ Mass
$<\sigma(*C_{CUP}), \mathbf{\{\sigma(*C_{CUP})\}}>$ Shift to singular count

the coffee in the pot $\rightarrow$ $<\sigma(*C_{POT}), *C_{POT}>$ Mass
$<\sigma(*C_{POT}), \mathbf{\{\sigma(*C_{POT})\}}>$ Shift to singular count

the coffee in the cup and the coffee in the pot $\rightarrow$ **sum** of singular count interpretations:

$<\sigma(*C_{CUP}) \sqcup \sigma(*C_{POT}), \{ \mathbf{\sigma(*C_{CUP})}, \mathbf{\sigma(*C_{POT})} \}>$ Plural count

$\mathbf{count}_{\{\sigma(*CCUP), \sigma(*CPOT)\}}(\sigma(*C_{CUP}) \sqcup \sigma(*C_{POT})) = \{\sigma(*C_{CUP}), \sigma(*C_{POT})\}$ (2 elements)

Both in (1) distributes to the two elements in the count-set.

Singular shift does not involve atomization, but does introduce grid.
Unlike group-formation, the first tier is without grid. Grid is encoded in the second tier.

Similar to Landman 2000's operation $\uparrow$ and his interpretation of collectivity as singularity.
Iceberg analysis of grid is close to Schwarzschild's covers (Schwarzschild 1996),
without the problems for Schwarzschild's covers (see eg. Landman 2000). So:

The voice is the voice of Landman, but the hands are the hands of Schwarzschild.

Gillon's problem (Gillon 1992, Chierchia 1998).

(2) The curtains and the carpets resemble each other.
(3) The drapery and the carpeting resemble each other

(3) only allows an interpretation in which drapery is compared with carpeting.
(2) also allows that reading, but it allows a variety of other reading, like:
-a 'distributive' reading where the curtains resemble each other and the carpets resemble each other.
-a 'cumulative' reading where anything resembling anything else counts, as long as it's a curtain or a carpet.

Challenge for Chierchia's theory: Chierchia assumes that the mass noun phrase *the carpeting* and the plural noun phrase *the carpets* have the same denotation.

Iceberg semantics:
I don't give the semantics for the predicate *resemble each other* but only the obvious felicity condition:

Felicity condition on *resemble each other*:
***each*:** *resemble each other* can only apply to **count objects**
other: *resemble each other* can only apply to objects with **at least two** counting parts

This felicity condition predicts the facts in (4):

(4) a. ✓The cats resemble each other
b. #The cat resembles each other
c. #The furniture resembles each other

(4a) <σ(*CAT), CAT>, is a **plural count object**. ✓
(4b) <σ(CAT), {σ(CAT)}>, is a singular count object with **only one counting part**. #
(4c) Mass: <σ(*MEUBEL),*MEUBEL> is a mass object with **no counting parts**. #
Collective: <σ(*MEUBEL),{σ(*MEUBEL)}> is singular (**also one counting part**) #

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(2) The curtains and the carpets resemble each other.

(2) has (at least) three felicitous interpretations:
1. Sum interpretation:
<σ(*CURTAIN) ⊔ σ(*CARPET), CURTAIN ∪ CARPET>
Resemble accesses counting set **CURTAIN ∪ CARPET**
This interpretation compares: curtains with curtains
curtains with carpets
carpets with curtains
carpets with carpets.

2. **Distributive interpretation:**
$\lambda P.\ P(\langle \sigma(^*\mathrm{CURTAIN}), \mathrm{CURTAIN}\rangle) \wedge P(\langle \sigma(^*\mathrm{CARPET}), \mathrm{CARPET}\rangle)$
This interpretation compares: curtains with curtains
carpets with carpets.

3. Sum of groups interpretation:
$\langle\, \sigma(^*\mathrm{CURTAIN}) \sqcup \sigma(^*\mathrm{CARPET}), \{\sigma(^*\mathrm{CURTAIN}), \sigma(^*\mathrm{CARPET})\}\rangle$
Resemble accesses counting set $\{\sigma(^*\mathrm{CURTAIN}), \sigma(^*\mathrm{CARPET})\}$ (2 elements)
This interpretation compares: curtains with carpets
carpets with curtains
This is the reading that (2) shares with (3).

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(3) The drapery and the carpeting resemble each other

-No sum interpretation
$\langle \sigma(^*\mathrm{CURTAIN}) \sqcup \sigma(^*\mathrm{CARPET}), {}^*\mathrm{CURTAIN} \cup {}^*\mathrm{CARPET}\rangle$
*CURTAIN $\cup$ *CARPET is not disjoint, so there is no counting set.

-No distributive interpretation
$\lambda P.\ P(\langle \sigma(^*\mathrm{CURTAIN}), {}^*\mathrm{CURTAIN}\rangle) \wedge P(\langle \sigma(^*\mathrm{CARPET}), {}^*\mathrm{CARPET}\rangle)$
*CURTAIN is not disjoint and neither is *CARPET, so there are no counting sets.

3. Sum of groups interpretation:
As in the case of the coffee above, singular shift creates a plural count interpretation:
$\langle\, \sigma(^*\mathrm{CURTAIN}) \sqcup \sigma(^*\mathrm{CARPET}), \{\sigma(^*\mathrm{CURTAIN}), \sigma(^*\mathrm{CARPET})\}\rangle$
Resemble accesses counting set $\{\sigma(^*\mathrm{CURTAIN}), \sigma(^*\mathrm{CARPET})\}$
This interpretation compares: curtains with carpets
carpets with curtains
This is the reading that (2) shares with (3).

More fine-grained singular shift requires more fine-grained Iceberg semantics:
Think of the second tier set as the top of a **second tier store:**

$x = \langle x^1, x^2[\ ]\rangle$

Fine-grained singular shift:
$\uparrow(x) = \langle x^1, \{x^1\}[x^2]\rangle$

Fine-grained singular shift keeps track of the original mass-count characteristics of the DP. This means, among others, that fine-grained singular shift allows an inverse operation, which is useful.

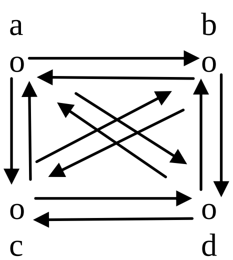
6. Iceberg semantics for modifiers

Iceberg semantics for complex NPs:
(5a): two modifiers: *vier* (*four*) and *elkaar bespionerende* (*spying on each other*).
The constituent structure in Dutch is as in (5c):

(5) a. De vier elkaar bespionerende wielrenners
The four each other spying on cyclists
b. The four cyclists spying on each other
c. [$_{DP}$ De [$_{NP}$ vier [$_{NP}$ [elkaar bespionerende] wielrenners]]]

The definedness conditions of the definite in (5a) forms a guide to the iceberg semantics for the modifiers. The definite in (5a) should be defined in the following two situations:

Situation 1:
We are in a bicycle match, one minute before the winner crosses the finish line. Four cyclists have escaped from the peloton and have left the peloton on a distance of fifteen minutes, hence without a chance for a position among the first three. The first four are nervous, the others are relaxed and taking their time.



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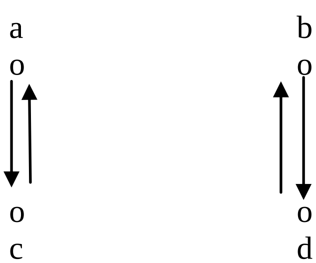
Situation 2:
The same situation, with the only difference that two of the four got away and will decide the race between them. Five minutes behind them, the other two will fight for the third place. The first four are nervous, the others are relaxed and taking their time.



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(5a) is defined in situation 1: there are four cyclists spying on each other.
(5a) is defined in situation 2: there are **twice two** cyclists spying on each other.
Since **twice two** is four, there are in situation (2) **all in all** four cyclists spying on each other.

NP

MOD — NP **[head]**

spying on each other — *cyclists*

> ***Head assumption*:**
> **The second tier semantics of the *head* constrains the first and second tier interpretation of the modifier structure.**

1. NP^1: First tier interpretation of the modifier structure

> **Head assumption:** The modifier maps the **two-tier interpretation of the head** onto a set interpretation.

As above, no analysis of the semantic content of *spying on each other*,
but implementation of the presuppositional content derived from *each other*.

spying on each other + N
-requires input *N* to be plural count
-restricts the first tier output interpretation to objects in N^1 that are **pluralities of singularities in N^2**

$$\lambda N\ \lambda x.\begin{cases} \textit{spying on each other}(x) \wedge N^1(x) & \textbf{if } N \textbf{ is plural and } \mathbf{card}_N^2(x) \geq 2 \\ \bot & \text{otherwise} \end{cases}$$

This maps *N* onto the set of plural sums of N^2 entities that are in N^1 and are spying on each other

2. NP^2: Second tier interpretation of the modifier structure.

> **Head assumption: NP^2 = second tier interpretation of the head as restricted by NP^1**
>
> $NP^2 = (NP^1] \cap N^2$

Assuming plausible details for the semantics of *spying on each other*, this derives:

elkaar bespionerende wielrenners $\rightarrow$ $NP = \langle NP^1, NP^2 \rangle$

Situation 1:
$NP^1 = \{a\sqcup b, a\sqcup c, a\sqcup d, b\sqcup c, b\sqcup d, c\sqcup d, a\sqcup b\sqcup c, a\sqcup b\sqcup d, a\sqcup c\sqcup d, b\sqcup c\sqcup d, a\sqcup b\sqcup c\sqcup d\}$
$NP^2 = \{a, b, c, d\}$

Situation 2:
$NP^1 = \{a\sqcup b, c\sqcup d, a\sqcup b\sqcup c\sqcup d\}$
$NP^2 = \{a, b, c, d\}$

NP^2 is the set of all singular cyclists that are part of a spying-cyclist-plurality.

Numerical modifiers: similar semantics + the same modification interpretation schema:

$$four \rightarrow \lambda N\, \lambda x. \begin{cases} N^1(x) \wedge \mathbf{card}_{N^2}(x) = 4 & \textbf{if } \boldsymbol{N} \textbf{ is count (in fact, plural)} \\ \perp & \text{otherwise} \end{cases}$$

This maps N onto the set of sums in N that have 4 parts that are N^2

NP = *four* cyclists

$NP^1 =$ $\lambda x.\ {}^*\text{CYCLIST}(x) \wedge \mathbf{card}_{CYCLIST}(x) = 4$
$NP^2 =$ $\text{CYCLIST} \cap (\lambda x.\ \mathbf{card}_{CYCLIST}(x) = 4] = \text{CYCLIST}$

Hence: **Numerical modifier restriction:**
Numerical modifiers can only modify count nouns, not mass nouns.

NP = four cyclists spying on each other

Situation 1 and situation 2:
$\boldsymbol{NP^1} =$ $\{a\sqcup b\sqcup c\sqcup d\}$
$\boldsymbol{NP^2} =$ $\{a, b, c, d\}$

Corollary:
The definite *the four cyclists spying on each other* is defined in situation 1 and in situation 2.

A remark on measure constructions.

Landman ms. *Iceberg semantics*:
Extension of the modifier semantics to measure constructions: *three kilos of books*

Assumption 1: *three kilos* is semantically a **modifier** of *books* (**with** Rothstein)
Assumption 2: *kilos*, and hence *three kilos* is the **head** of the measure construction (in English and Dutch, **against** Rothstein)
Analysis **3:** I present a semantics of the measure head as a **mass function**
4: As in the present section, the modifiction semantics obeys the **Head Assumption**.
Corollary **4:** *Three kilos of books* is shown to ea **mass NP** (**with** Rothstein)

7. Partitive noun phrases

(6) a. Three of the cats
b. $[_{DP}$ *three* $[_{NP}$ *of* $[_{DP}$ the cats]]

Two assumptions that are important here:
- *of the cats* is an NP with NP semantics.
- NP forming operation *of* is ambiguous between Partitive operations **of** and **of***:

Partitive operations: of and **of***:

$$\textbf{of} \;=\; \lambda x.\begin{cases} <\boldsymbol{(}x^{1}\boldsymbol{]} \cap x^{2}, x^{2}> & \text{if } x \text{ is not singular} \\ \bot & \text{otherwise} \end{cases}$$

$$\textbf{of}^{*} \;=\; \begin{cases} <\boldsymbol{(}x^{1}\boldsymbol{]} \cap {}^{*}x^{2}, x^{2}> & \text{if } x \text{ is not singular} \\ \lambda x. & \\ \bot & \text{otherwise} \end{cases}$$

The semantics effects of these operations are shown below for plural DP complements.

The semantics of the partitive operations builds in a stipulation:
Partitive with singular DP complement is undefined: , *#one of the cat*

Fission = splitting the atom = "grinding" singular count objects into (neat) mass objects

Fission: Let x be a single count object. (Landman 2011)
$\downarrow(x) = <x^1, \boldsymbol{(}x^1\boldsymbol{]}>$

Fission accesses the parts of x^1 that are below the iceberg

Fission shift a **singular count object** *fido* = <fido, {fido}> into a **(neat) mass object** by replacing the second tier interpretation {fido} by the set of all fido's parts, $\boldsymbol{(}$fido$\boldsymbol{]}$.

Fission as a rescue mechanism: Partitive with singular DP complement can be **rescued** by shifting the singular DP complement with fission.

Consequence: Partitive with singular DP complement behaves like Partitive with mass DP complement.

The difference between **of** and **of*** is only relevant for partitives with plural DP complements:
-Singular DP complements are stipulated to be infelicitous.
-In neat mass DP complements x^2 is itself already pluralized and $^{**}x^2 = {}^{*}x^2$.

Mass DPs:

of the furniture $\rightarrow$

$\textbf{of}(\langle\sigma(\text{*MEUBEL}), \text{*MEUBEL}\rangle) = \langle \boldsymbol{(}\sigma\boldsymbol{(}\text{*MEUBEL})\boldsymbol{]} \cap \text{*MEUBEL}, \text{*MEUBEL}\rangle$
$= \langle\text{*MEUBEL}, \text{*MEUBEL}\rangle$

So: ***of the furniture*** and ***furniture*** have the same denotation.

This means that ***of the furniture*** is **mass**:

#three of the furniture
#each of the furniture
✓*much of the furniture*

Singular DPs + fission:

of the cat $\rightarrow$

$\textbf{of} + \langle\sigma(\text{CAT}), \{\ \sigma(\text{CAT})\}\rangle \;=\; \textbf{[fission]} \quad \textbf{of}(\downarrow(\langle\sigma(\text{CAT}), \{\ \sigma(\text{CAT})\}\rangle)) \quad =$
$= \quad \textbf{of}(\langle\sigma(\text{CAT}), \boldsymbol{(}\sigma(\text{CAT})\boldsymbol{]}\rangle) \quad = \quad \langle\boldsymbol{(}\sigma(\text{CAT})\boldsymbol{]}, \boldsymbol{(}\sigma(\text{CAT})\boldsymbol{]}\rangle$

The partitive with singular DP complement *of the cat* is rescued with fission;
The **fission interpretation of *of the cat*** is **mass**:

#three of the cat
#each of the cat
✓*much of the cat*

(7) After the kindergarten party, *much of my daughter* was covered with paint.

Partitives with plural DP complements: two interpretations.

of the cats $\rightarrow$ $\quad \textbf{of}\ (\langle\sigma(\text{*CAT}), \text{CAT}\rangle)$
$\rightarrow \quad \textbf{of}^*(\langle\sigma(\text{*CAT}), \text{CAT}\rangle)$

1. $\textbf{of}(\langle\sigma(\text{*CAT}), \text{CAT}\rangle) \quad = \quad \langle\boldsymbol{(}\sigma(\text{*CAT})\boldsymbol{]} \cap \text{CAT}, \text{CAT}\rangle \quad = \quad \langle\text{CAT}, \text{CAT})\rangle$

Partitive operation **of:** *of the cats* $\rightarrow$ $\langle\text{CAT}, \text{CAT}\rangle$ **singular count noun**

2. $\textbf{of}^*(\langle\sigma(\text{*CAT}), \text{CAT}\rangle) \quad = \quad \langle\boldsymbol{(}\sigma(\text{*CAT})\boldsymbol{]} \cap \textbf{*CAT}, \text{CAT}\rangle \quad = \quad \langle\text{*CAT}, \text{CAT})\rangle$

Partitive operation **of*:** *of the cats* $\rightarrow$ $\langle\text{*CAT}, \text{CAT}\rangle$ **plural count noun**

Partitive with plural DP complement is compatible ***both*** with singular and plural determiners:

✓*one of the cats*
✓*three of the cats*
✓*each of the cats*
✓*most of the cats*

What about fission readings of plural DPs?

Potential third reading, namely the singular shift reading of the plural DP:

$$\mathbf{of} + \uparrow(<\sigma(*CAT), *CAT>)) \quad = \quad \mathbf{of} + <\sigma(*CAT), \{\sigma(*CAT)\}>$$

The complement is a singular DP, so applying **of** is infelicitous.
If we allow fission, we get a mass interpretation for *of the cats* analogous to that of the singular predicate *of the cat*:

of the cats $\rightarrow$ $<\boldsymbol{(}\sigma(*CAT)\boldsymbol{]}, \boldsymbol{(}\sigma(*CAT)\boldsymbol{]}>$ (neat mass interpretation)

Not everybody seems to accept fission interpretations for partitives with plural complements. Rothstein p.c. accepts (7), but doesn't accept (8a) (while (8b), of course, only has a count reading).

(7) After the kindergarten party, *much of my daughter* was covered with paint.
(8) a. #After the kindergarten party, *much of the kids was* covered with paint.
b. After the kindergarten party, *many of the kids were* covered with paint.

Possibly the combination of singular shift and fission shift is too much for her.
Surveying the internet, it seems that other speakers have less problems with accepting cases like (8a).
A sample (only examples where the context makes it reasonably clear that the author is a native speaker of English, all examples (re-)accessed 9-9-2013):

(9) a. Today, thanks to the restoration efforts, ***much of the walls*** and the four gates still stand. *http://en.wikipedia.org/wiki/Geumjeongsanseong*
b. While our current sensibilities are accustomed to the tans, taupes, grays and browns, in their time ***much of the rooms*** as well as the cathedral proper would have been beautifully painted. http://ksamsontheroad.blogspot.co.il/
d. (…)forcing his daughter to spend ***much of the days before his death*** driving around trying to fill morphine prescriptions.
http://www.nursingtimes.net/nursing-practice/clinical-zones/older-people/nhs-failing-older-people-ombudsman-reports/5025756.article
e. As we've watched their characteristics and looks emerge, Anthony and I have wondered ***how much of the children*** is down to the genes they've inherited from us - and how much is down to fate and the environment.
http://www.dailymail.co.uk/health/article-2005953/Genetic-testing-children-predict-future-health.html

(the same point with *how much*: *how much of the children* is **mass**.)

8. Quantitative comparison and dimensions.

Semantics of *most* applied to partitives with fission-shifted singular DP complements.

(10) Most of the house is painted green.
(11) a. Most of the artichoke is inedible.
b Most of this artichoke was quite inedible.

Fission shift interpretations: *of the house* $\rightarrow$ <**(**σ(HOUSE)**]**, **(**σ(HOUSE)**]** >
Fission shift accesses the parts that are below the iceberg. They form the input for the mass measure.

(10) Most ***of the house*** is painted green.
most[$\lambda x.x \sqsubseteq \sigma(house)$, *green*] Mass interpretation of **most**

***Most* for count predicates:** what *most* ranges over is open to contextual manipulation, but the extent of the manipulation involved is restricted.

(12) Most US citizens voted for Obama
US citizens = US citizens who voted

most for mass predicates: much more serious context dependency:
mass measures may not even be applicable unless contextual restriction takes place.

Contextual restriction via a **homogenous restriction dimension**.

Homogeneity. Let P be a property, $d \in B$.
d **is homogeneously** P iff P(d) and for every d' v d: P(d') or P(d')=$\perp$
(If you are homogeneously yellow, you may have parts that are too small to count as yellow, but you don't have blue parts.)

homP = {$d \in D$: d is homogeneously P}
P restricted to homogeneous object

The restriction dimension based on P: $\delta_P = \sigma(\mathbf{hom}P)$
The homogeneous sum of objects hat have P homogeneously

-The notion of homogeneity excludes as dimensions object that are " too big" wrt to P:
ie. if a wall is half yellow, half red, it is not yellow, but not homogeneously not-yellow.
We only want to compare the yellow part with the homogeneously non-yellow part.

A **dimensional measure function μ_P** selects a restriction dimension δ_P **for which μ is defined**, and assigns to each object d, the value of $d \sqcap \delta_P$

(10) Most of the house is painted green.

- **area(**σ(HOUSE) **=** $\bot$**.** We don't know what the area of a three dimensional object is.
-P = *is part of outside walls* (a homogeneous property)
- if $x \sqsubseteq \sigma(\text{HOUSE})$, then $\mathbf{area}_P(x) = \mathbf{area}(x \sqcap \delta_P)$
So: the area of the house is the area of the outside walls of the house.

---

(11) a. Most of the artichoke is inedible.

-**weight** or **volume** are applicable measures

---

(11) b. Most of this artichoke was quite inedible.

- **weight** or **volume** stay applicable measures.
But in context, a dimension $\delta_{\text{generally edible}}$ is added, and the measure is:
$\mathbf{weight}_{\text{generally edible}}$ or $\mathbf{volume}_{\text{generally edible}}$

Dimensionally restricted semantics of *most*:

most[X, Y] is true rel. μ and δ_P iff

$$\mu_P(\ \sigma(\mathbf{hom}(X \cap Y)\ ) > \mu_P(\sigma(X) - \sigma(\mathbf{hom}(X \cap Y)\ )$$

The semantics is formulated in terms of relative complement:
you compare in (10): *the **homogeneously** green part of the house* and
*the **homogeneously** non-green part of the house*

We assume that *green* is homogenous. (10) is true iff

$$\mathbf{area}_P(\ \sigma(\lambda x.x \sqsubseteq \sigma(\mathit{house}) \wedge \mathit{green}(x))\ ) >$$
$$\mathbf{area}_P(\ \sigma(\mathit{house}) - \sigma(\lambda x.x \sqsubseteq \sigma(\mathit{house}) \wedge \mathit{green}(x))\ )$$

(10) is true iff ***of the outside surface area of the house***
the area of the green part is larger than the **relative complement area** (the area that doesn't overlap any green part)

---

(11) a. Most of the artichoke is inedible.

No dimensional restriction. (11a) is true iff
the volume of the homogeneously inedible part of the artichoke is larger than the volume of the homogeneously edible part.

b Most of this artichoke was inedible.

Dimensional restriction: $\delta_{\text{generally edible}}$. (11b) is true iff
the volume of the generally edible part which this time was inedible is larger than the volume of the generally edible part which this time too was edible.

Conclusion

Iceberg semantics is a fruitful framework for developing, comparing, and simplifying theories of singular count, plural count, neat mass and mess mass nouns, extending the notions that are usually applied only to lexical nouns, to complex nouns (NPs) and noun phrases (DPs).

Appendix 1. The operation of externalization Δ.

Count noun interpretation: set of generators is required to be disjoint **in context**.
(Interpretations are always contextually restricted)

The count or mass nature of a noun is not an empirical finding but a grammatical choice.
NOT: Generator set X happens not to be disjoint, so the noun turns out to be mass. Hurray!
BUT: If the contextual denotation chosen for a count noun has a set of generators which is not disjoint, the semantic system must do something to avoid a crash.

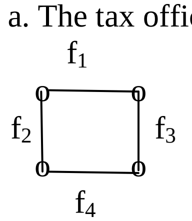
The crash must be avoided for count nouns and for count DPs.

Rothstein 2010: discussion of nouns like *fence*:
Most easily available technique for making a set of generators disjoint: contextual restriction.
A fence may be made up of four sub-fences, but the context will make it clear whether it counts as one fence or four.
So, in context you kick out the salient denotation of *fence* the offending sub-fences (or the super fence).

Externalization: shifting operation that can be applied to when this doesn't help.

This is required when the generators overlap, but we stubbornly decide to count anyway, or to distribute anyway:

(13) a. The tax office decided that the farmers had put up 13 taxable fences.



f_1, f_2, f_3, f_4
$f_1 \sqcup f_2,\ f_2 \sqcup f_3,\ f_3 \sqcup f_4,\ f_4 \sqcup f_1$
$f_1 \sqcup f_2 \sqcup f_3,\ f_2 \sqcup f_3 \sqcup f_4,\ f_3 \sqcup f_4 \sqcup f_1,\ f_4 \sqcup f_1 \sqcup f_2,$
$f_1 \sqcup f_2 \sqcup f_3 \sqcup f_4$

b. My hand and its five fingers are six body parts.

Externalization, Δ, maps any object - singular, plural or mass – onto a 'version' of that object, a 'perspective' on that object which is not in danger of overlap problems.

Δ maps my hand onto an object Δ(my hand) such that:
while my hand ⊓ my thumb ≠ 0, Δ(my hand) ⊓ my thumb = 0

An equivalence relation, ~, of intensional identity expresses that though my hand and Δ(my hand) are distinct elements of B, they are *intensionally identical*.

Implementation:
We assume that **B** contains what I will call a Δ-*domain*, which consists of two sets, I (for interior) and E (for exterior) with Δ a bijection from I to E.

Idea: semantic interpretations are, by default, taken from I.
If a set that ought to be disjoint is not disjoint in I, we can *freely* switch internal object x to the equivalent external object Δ(x) and resolve the crash.

The external set is a disjoint set external to I:

A Δ-d**omain** in **B** is a tuple <Δ,**I**,E**, ~**> where:

1. **I** $\subseteq$ **B i**s a complete atomic a Boolean algebra with the same 0 as B.
2. E $\subseteq$ B is a set such that no element of E overlaps any other element of I $\cup$ E.
3. Δ is a one-one function from I into E.
4. ~ is an equivalence relation on **B** such that for all x $\in$ I: x ~ Δ(x)

We assume: **B** contains a Δ-domain <Δ,**I**,E**, ~** >.

body part is a count noun
In context we can include simultaneously: h (my hand) and f_1, f_2, f_3, f_4, f_5 (my five fingers)
To avoid the crash we shift to Δ(h): no overlap is involved.

Similarly for *my hand and its five fingers*:

$$< \Delta(h) \sqcup f_1 \sqcup f_2 \sqcup f_3 \sqcup f_4 \sqcup f_5, \mathbf{\{ \Delta(h), f_1, f_2, f_3, f_4, f_5\}} >$$

Moral: Examples **can** be found of count nouns where we **are** counting overlapping elements. The unnaturalness of these cases is in fact an argument in favor of the disjointness requirement on the generator sets of count nouns.
See further, lots of discussion in Rothstein 2010, 2011.

Appendix 2. Neat mass nouns.

Count nouns: A plurality of *cats* does not itself count as one *cat*.
One *group of cats* maybe, but not one *cat*.
Neat mass nouns: A plurality of kitchenware, like *the cup and saucer* counts as **one item** along side its parts *the cup* (**one item**) and *the saucer* (**one item**).

Idea: *neat nouns* denote are sets in which the distinction between *singular individuals* and *plural individuals* is not properly articulated.

Barner and Snedeker 2005: neat mass nouns pattern with count nouns in easily available count comparison readings. e.g. (14a) and (14b) are equivalent (examples based on Landman 2011):

(14) a. Op deze boerderij is *het meeste vee* buiten in de zomer.
On this farm, *most livestock* is outside in summer.
b. Op deze boerderij zijn de meeste boerderij dieren buiten in de zomer.
On this farm, *most farm animals* are outside in summer

Landman 2011: neat mass nouns **also** pattern with mess mass nouns
unlike count nouns, neat mass nouns allow mass measures in *most*.

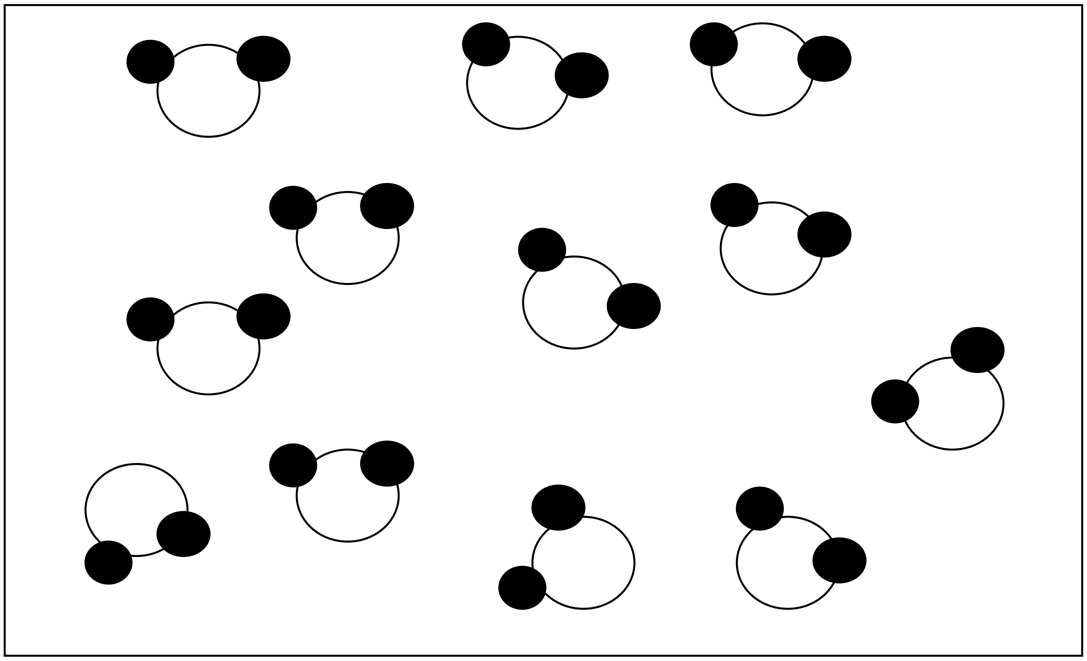
Our live stock is cows and chickens. The cows are outside, the chickens inside.
The chickens outnumber the cows, but there is less biomass and less volume of chicken.

(15) a. Wat biomassa betreft/In termen van volume wordt *het meeste vee* buiten gehouden
With respect to biomass/In terms of volume, *most live stock* is kept outside.
b. #Wat biomassa betreft/In termen van volume worden *de meeste stuks vee* buiten gehouden.
#With respect to biomass/In terms of volume, *most items of livestock* are kept outside.

Neat mass nouns: (15a) is true and felicitous. Hence mass comparison for neat mass nouns.
Count nouns: (15b) is false and a bit weird (what is the adjunct doing there). Count comparison.

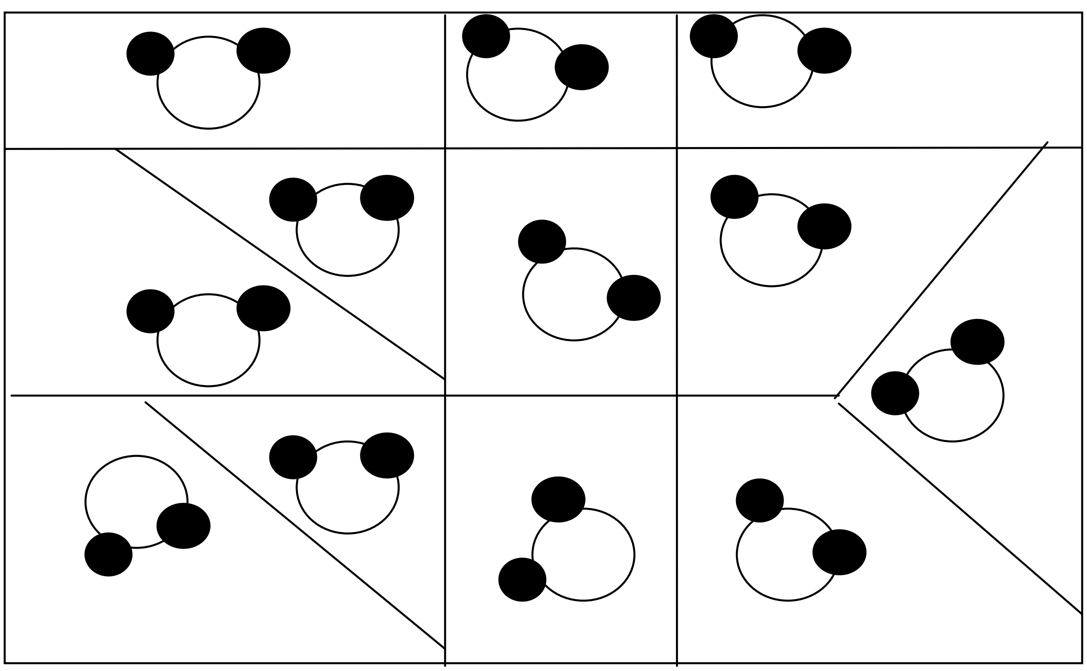
Appendix 3. Mess mass nouns.

Prototypical mass noun: *water*.
The water in the container consists of Mickey Mouse Molecules of H_2O:



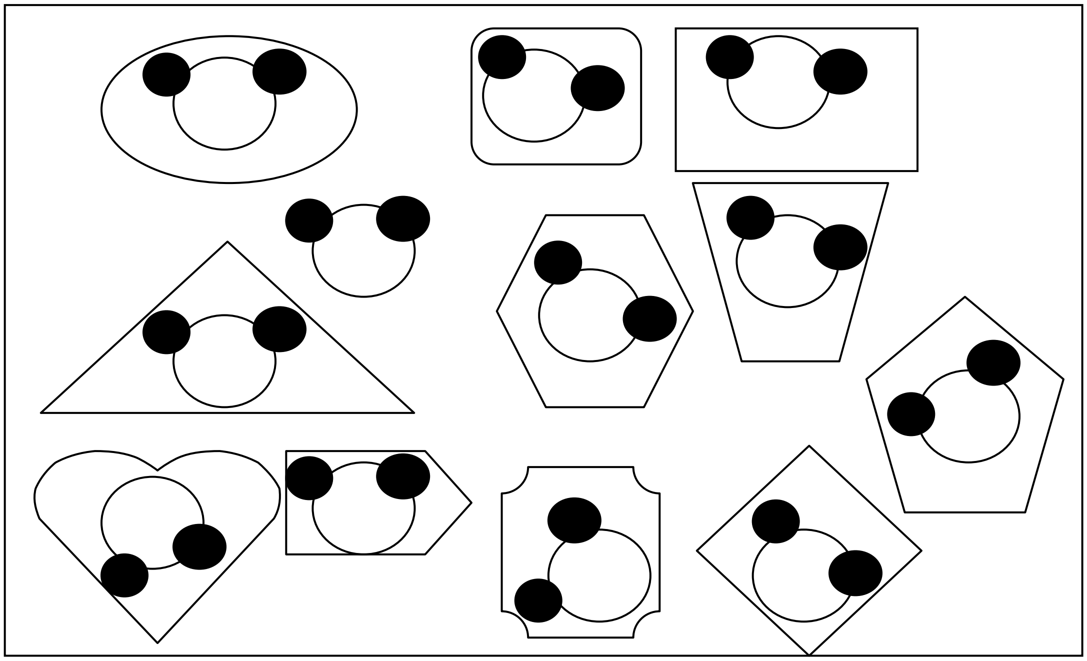
However, the actual substance in front of you is *not just* the water molecules, *but also the space* inside them and around them.
Identifying the water with just the molecules is a count perspective.

As a mass object, we **still *can*** partition the water into minimal parts, for instance, like this:



Each block of the partition is minimal in a counting sense: it is *water*, but cannot be split any more into **two** blocks that **each** count as *water*: i.e. if we **were** to assign a count, they **would** count as 1).

Problem: *the water cum space* can be partitioned into minimal parts in countlessly many ways, because the space can be divided up differently at will:



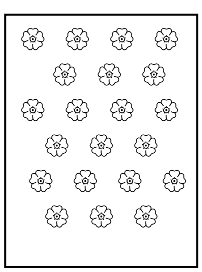
And none of these ways is privileged in any way.

I call one contextual partition of the water into minimal parts **a variant.**

The mess mass assumption: the water is generated from all contextually relevant variants.

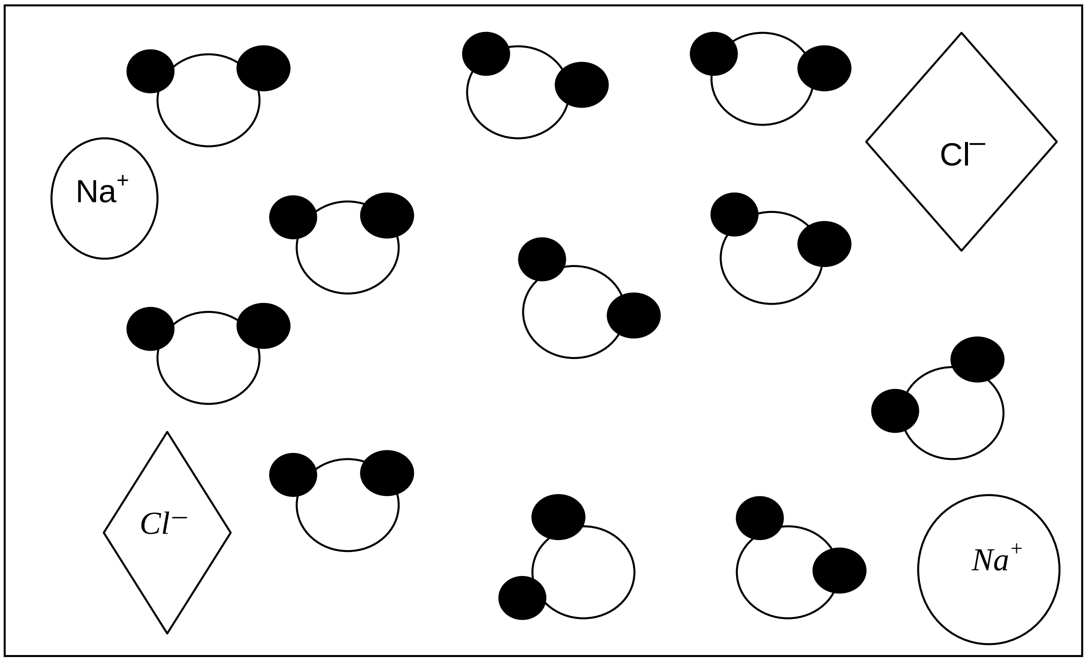
Hence: the denotation of the mess mass noun water is built from a **non-disjoint set of minimal parts of water.**
If you count, you count wrong, because you count **overlapping** generators.

Similar: *bloemetjes behang, flower patterned wall paper*



Other ways of overlap: *salt* dissolved in water:

(16) There is *salt* in the water, two molecules worth.



The *salt* is built from two salt molecules. But which ones?
NaCl + NaCl or *Na*Cl + Na*Cl* ?
The mess-mass assumption: The salt is built from **both variants simultaneously**.
If you count, you count wrong, because you count overlapping molecules.

This is the inspiration for the model. More prototypical (from Landman 2011):

Take a big juicy slab of meat. With Chierchia 1978, I think that we can think of this as being built from minimal parts. Not natural meat-parts, but minimal parts that are appropriately minimal in a context. For instance, they are the pieces as small as a skilled butcher, or our special fine-grained meat-cutting machine can cut them. Suppose the meat cutting machine consists of two sharp knife-lattices that cut the meat from left to right, and then from front to back, snap-snap. This will cut the meat into very many minimal meat pieces.

But if I move the knife-lattices slightly, the front-back knife to the left, the left-right knife to the front, and cut snap-snap, I get a different partition into minimal meat pieces. And of course, there has minimal meat pieces. None of these partitions has a privileged status, and none of these partitions provides *its* minimal pieces with the privileged status of being the 'real' minimal pieces. On my view, all of these pieces count equally as minimal meat pieces in the context given, and the meat is built from all of them.

Mess mass nouns:
Built from overlapping generators coming from a multiplicity of simultaneous variants, different ways of dividing the stuff into minimal parts.

Neat mass nouns: Built from disjoint minimal generators, but overlapping generators: distinction between singulars and plurals not articulated.

Count nouns: built from disjoint generators.

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