The Epistemology of Conditionals

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Overview

Why an epistemology of conditionals?

Two “certainties”

Adams’ Thesis

Intermezzo: experimental work on Stalnaker’s Hypothesis challenged

Triviality undone?

What’s the upshot?
Why an epistemology of conditionals? (I)

NB: We’ll only consider indicative conditionals.

Two seemingly uncontentious epistemological facts about conditionals:

1. Conditionals can be probable / assertable / acceptable to a greater or lesser extent.
2. Some of the information we receive is conditional in form, and such conditional information may impact out belief state.
Why an epistemology of conditionals? (II)

We have probability theory, purportedly general accounts of assertability as well as purportedly general accounts of acceptability, and also purportedly general accounts of belief change.

Don’t these apply to conditionals as well? If they do, why would we need an epistemology of conditionals?

Claim: there are reasons to believe that those purportedly general theories do not apply to conditionals, and that we do need an epistemology of conditionals.
Probability theory tells us that

- $\Pr(\neg \varphi) = 1 - \Pr(\varphi)$.
- $\Pr(\varphi \land \psi) = \Pr(\varphi) \Pr(\psi | \varphi)$.
- $\Pr(\varphi \lor \psi) = \Pr(\varphi) + \Pr(\psi) - \Pr(\varphi \land \psi)$.

But what does it say about $\Pr(\varphi \rightarrow \psi)$? That depends on how we are going to interpret $\rightarrow$.

Proposal: our interpretation of $\rightarrow$ is to be such that it validates Stalnaker’s Hypothesis, that is, $\Pr(\varphi \rightarrow \psi) = \Pr(\psi | \varphi)$. 
Assertability

Two accounts of assertion:

▶ Knowledge Account: one must assert $\varphi$ only if one knows $\varphi$.
▶ Rational Credibility Account: one must assert $\varphi$ only if $\varphi$ is rationally credible to one.

If conditionals do not express propositions, then they cannot be known, nor can they be rationally believed (to be true).

Nor do conditionals only require special treatment when it comes to stating assertability conditions if they do not express propositions.
Acceptability

- There is widespread agreement that high probability is close to being sufficient for rational credibility (NB: Kyburg’s lottery paradox).

- Suppose conditionals express the proposition expressed by the corresponding material conditional.

- Then a conditional is highly probable if its antecedent is highly improbable.

- But consider this:

  (1) If Manchester United ends last in this year’s Premier League, they will shoot their coach.

  Although it is exceedingly unlikely that Manchester United will end last in this year’s Premier League, we do not find (1) rationally credible.

- This suggests that, assuming the material conditional account, high probability is not even nearly correct as a sufficient condition for the rational credibility of conditionals.
Belief change (I)

How do we accommodate the receipt of information of the following type?

- If it continues to rain, then tomorrow’s match will be cancelled.

- If the emission of greenhouse gases does not diminish, then we should expect desertification of parts of Europe.

This question has been considered to some extent by computer scientists (e.g., Boutillier, Goldschmidt, Kern-Isberner), but only from the perspective of AGM theory.
Belief change (II)

Next to the epistemology of belief (which AGM theory is concerned with), there is also an epistemology of degrees of belief.

According to Bayesian epistemology (the main epistemology of degrees of belief), we accommodate new information by means of Bayes’ rule, aka conditionalization.

Skyrms in 1980: “we have no clear conception of what it might be to conditionalize on a conditional.”

Seems still true today!

Two “certainties”

We consider only the static part of the epistemology of conditionals.

In this part, there are two “certainties” to report (everything else is highly controversial):

1. Stalnaker’s Hypothesis (SH) is false.
2. Adams’ Thesis (AT) is true.

Claim: 1 is questionable (SH may well be true); 2 is false (AT is false).
Triviality

Why do all believe that SH is false?

Take any Pr for which SH holds, and let \( \varphi \) and \( \psi \) be such that both \( \Pr(\varphi \land \psi) > 0 \) and \( \Pr(\varphi \land \neg \psi) > 0 \). Then by the law of total probability,

\[
\Pr(\varphi \rightarrow \psi) = \Pr(\varphi \rightarrow \psi \mid \psi) \Pr(\psi) + \Pr(\varphi \rightarrow \psi \mid \neg \psi) \Pr(\neg \psi),
\]

which, using SH, can be rewritten as

\[
\Pr(\varphi \rightarrow \psi) = \Pr(\psi \mid \varphi \land \psi) \Pr(\psi) + \Pr(\psi \mid \varphi \land \neg \psi) \Pr(\neg \psi),
\]

which further simplifies to

\[
\Pr(\varphi \rightarrow \psi) = 1 \times \Pr(\psi) + 0 \times \Pr(\neg \psi).
\]

Apply SH again to see how strange this is!
Adams’ Thesis

If one wants to save at least the gist of SH—which, after all, does sound right—then the triviality results give one grounds for denying that conditionals express propositions: if conditionals do not express propositions, then various of the conditional probabilities figuring in the results are not well-defined.

What remains of SH once truth-conditionality is given up?

Ernest Adams proposed to restrict SH to simple conditionals and to interpret \( \Pr(\varphi \rightarrow \psi) \) not as the probability of the conditional but of its acceptability (originally: assertability). This is now called “Adams’ Thesis” (AT), and sometimes written as:

\[
\text{Acc}(\varphi \rightarrow \psi) = \Pr(\psi | \varphi).
\]
Adams’ Thesis: “evidence” for descriptive adequacy

Virtually all philosophers believe that AT is descriptively adequate:

Jackson: “There is a great deal of evidence for AT. There is head-counting evidence. Very many philosophers of otherwise differing opinions have found AT highly intuitive. There is case-by-case evidence. Take a conditional which is highly assertible . . . ; it will invariably be one whose consequent is highly probable given its antecedent.”

McGee: “AT describes what English speakers assert and accept with unfailing accuracy . . .”

Bradley: “AT is massively supported by the empirical evidence relating to our actual use of indicative conditionals.”
The Adams Family

Often in the literature, Adams’ proposal is stated as follows: the acceptability of a conditional goes by the corresponding conditional probability.

Explications of this idea less strict than (AT)

\[ \text{Acc}(\varphi \rightarrow \psi) \approx \Pr(\psi \mid \varphi). \] (WAT1)

\[ \text{Acc}(\varphi \rightarrow \psi) \text{ is high/middling/low iff } \Pr(\psi \mid \varphi) \text{ is high/middling/low}. \] (WAT2)

\[ \text{Acc}(\varphi \rightarrow \psi) \text{ highly correlates with } \Pr(\psi \mid \varphi). \] (WAT3)

\[ \text{Acc}(\varphi \rightarrow \psi) \text{ at least moderately correlates with } \Pr(\psi \mid \varphi). \] (WAT4)

NB: For each of these, the same provisos hold as for AT.
Experimental work on Adams’ proposal (I)

Douven and Verbrugge [2010] makes the following proposal:

Proposal

Group so-called inferential conditionals according to the type of inference they reflect. So,

\[ \varphi \rightarrow \psi \text{ is a deductive inferential (DI)/inductive inferential (II)/abductive inferential (AI) conditional iff } \psi \text{ is a deductive/inductive/abductive consequence of } \varphi. \]

These have corresponding contextual versions:

\[ \varphi \rightarrow \psi \text{ is a contextual DI/II/AI conditional iff } \psi \text{ is a deductive/inductive/abductive consequence of } \{\varphi, \varphi_1, \ldots, \varphi_n\}, \text{ with } \varphi_1, \ldots, \varphi_n \text{ being background premises salient in the context in which } \varphi \rightarrow \psi \text{ is asserted or being evaluated.} \]
The typology was used to test Adams’ proposal.

In particular, their experiments were meant

1. to investigate relationship between acceptability of inferential conditionals and corresponding conditional probabilities;
2. to investigate whether there are significant differences for the different types of inferential conditionals between acceptability and probability ratings.
Experiment: example (II conditional)

**Context:** According to a recent report written on the authority of the Dutch government, many primary school students in the province of Friesland (where many people still mainly speak Frisian) have difficulty with spelling. Jitske is a student of a primary school somewhere in the Netherlands.

**Conditional:** If Jitske goes to a Frisian primary school, then she has difficulty with spelling.

Indicate how acceptable you find this conditional in the given context:
Highly unacceptable  1  2  3  4  5  6  7  Highly acceptable
Experiment: results

1. **Negative** results: refutation of AT, WAT1, and WAT2.

2. **Positive** result: the acceptability of conditionals highly correlates with their corresponding conditional probabilities when taken over all inferential conditionals; so WAT3 is supported.

3. Per type of conditional:
   - AT holds only for DI conditionals.
   - For AI conditionals, acceptability goes strictly by conditional probability: WAT3 holds for them.
   - For II conditionals, not even this is true. At best, the acceptability of these conditionals can be said to go loosely by conditional probability: the correlation between acceptability and conditional probability is only moderately high.
A perplexing situation (I)

- Almost all philosophers believe that Adams was right.
- This judgment is based mainly on “intuition.”
- The empirical results show Adams to have been wrong.
- Almost all philosophers believe SH to be wrong.
- This judgment is based on formal results (the triviality arguments).

But ... empirical work by Evans, Over, and other experimental psychologists seems to indicate that SH is at least descriptively correct!
A perplexing situation (II)

How to reconcile the empirical results on SH with the triviality arguments?

Evans and Over [2004]: there is noise on the empirical data, and in view of the triviality arguments SH could still hold approximately.

As Hájek and Hall [1994] had already shown, if the triviality arguments undermine SH, then they also undermine the “approximate” version of SH, \( \Pr(\varphi \rightarrow \psi) \approx \Pr(\psi | \varphi) \).

Suggestion: reconsider the triviality arguments.
Experimental work on SH challenged (I)

Johnson-Laird and various co-workers have challenged the experimental results about SH.

Claim
The probability operator takes narrow scope over a conditional, so that asking how probable a given conditional is amounts to asking for the corresponding conditional probability.

People interpret

\[ \text{How probable is } \psi \text{ if } \varphi? \]

as

\[ [\text{How probable is } \psi ] \text{ if } \varphi? \]

and not as

\[ \text{How probable is } [\psi \text{ if } \varphi]? \]
Experimental work on SH challenged (II)

Johnson-Laird et al. support their claim by pointing at how we tend to interpret negated conditionals: it’s not the case that if $\varphi$, $\psi$, is typically interpreted as meaning that if $\varphi$, then it’s not the case that $\psi$.

Politzer, Over, and Baratgin [2010]: but not all operators take narrow scope over conditionals. E.g., we do not interpret

$$\text{It’s necessarily the case that if } \varphi, \text{ then } \varphi.$$  

as

$$\text{If } \varphi, \text{ then it’s necessarily the case that } \varphi.$$  

Correct – but still leaves the question unsettled whether the probability operator sides with the negation operator or with the necessity operator insofar as the scope issue is concerned.
Consider operators that are conceptually close to probability: acceptability, plausibility, credibility, assertability.

We can make sense of conditional acceptability, conditional plausibility, etc., via a generalized version of the Ramsey test (which is commonly regarded as an operational definition of conditional probability):

1. Add $\psi$ hypothetically to your stock of beliefs.
2. Make adjustments to maintain consistency (if necessary).
3. Ask yourself how $X$ (acceptable, plausible, etc.) $\varphi$ is in the resulting belief state.

The result gives you the conditional $X$-ability of $\psi$ given $\varphi$. 

If probability takes narrow scope over conditionals, then one would expect that closely related operators like acceptability, plausibility, etc., take narrow scope as well.

So, prediction: asking for acceptability (plausibility, etc.) of “If $\varphi, \psi$” yields same result as asking for conditional acceptability (etc.) of $\psi$ given $\varphi$.

Results are in for acceptability: there was a statistically significant difference between conditional acceptability ratings and acceptability ratings of the corresponding conditionals!

So acceptability does not take narrow scope over conditionals.
Triviality revisited (I)

In all of Lewis’ triviality arguments, we move from

\[ \Pr(\varphi \rightarrow \psi) = \Pr(\varphi \rightarrow \psi \mid \psi) \Pr(\psi) + \Pr(\varphi \rightarrow \psi \mid \neg \psi) \Pr(\neg \psi) \]

to

\[ \Pr(\varphi \rightarrow \psi) = \Pr(\psi \mid \varphi \land \psi) \Pr(\psi) + \Pr(\psi \mid \varphi \land \neg \psi) \Pr(\neg \psi). \]

Note that this move relies on something stronger than SH, namely, on

\[ \Pr(\varphi \rightarrow \psi \mid \chi) = \Pr(\psi \mid \varphi \land \chi). \quad \text{(GSH)} \]

From GSH we obtain SH (set \( \chi = \top \)) but not vice versa.

While SH has been extensively tested, experimentalists have so far ignored GSH.
Triviality revisited (II)

Some triviality arguments do not seem to rely on GSH but rather on SH together with

**IMPORT–EXPORT (IE)** “If \( \varphi \), then if \( \psi \), then \( \chi \)” and “If \( \varphi \) and \( \psi \), then \( \chi \)” are logically equivalent.

This sounds intuitively right.

Compare:

1. If he hears my arguments, then if he does not change his mind, he will leave the department.
2. If he hears my arguments and does not change his mind, he will leave the department.
Triviality revisited (III)

By IE and probability theory it holds that

\[ Pr(\text{If } \varphi, \text{ then if } \psi, \text{ then } \chi) = Pr(\text{If } \varphi \text{ and } \psi, \text{ then } \chi). \]

Thus, in particular,

\[ Pr(\text{If } \neg \psi, \text{ then if } \varphi, \text{ then } \psi) = Pr(\text{If } \varphi \text{ and } \neg \psi, \text{ then } \psi). \]

Applying SH to both sides of this equation yields

\[ Pr(\text{If } \varphi, \text{ then } \psi | \neg \psi) = Pr(\psi | \varphi \text{ and } \neg \psi) = 0. \]

NB: \( Pr(\text{If } \varphi, \text{ then } \psi | \neg \psi) = 0 \) iff \( Pr(\neg \psi | \text{If } \varphi, \text{ then } \psi) = 0 \) iff \( Pr(\psi | \text{If } \varphi, \text{ then } \psi) = 1. \)

This is absurd: it means that one cannot be certain of a conditional without being certain of the conditional’s consequent
Triviality revisited (IV)

But if SH is assumed, then GSH follows from IE.

1 follows from IE by the fact that probability theory respects logic; apply SH to both sides of 1 to obtain 2:

1. \( \Pr(\varphi \rightarrow (\psi \rightarrow \chi)) = \Pr((\varphi \land \psi) \rightarrow \chi) \).

2. \( \Pr(\psi \rightarrow \chi \mid \varphi) = \Pr(\chi \mid \varphi \land \psi) \).

So we only have to consider GSH.
**Context:** Judy is waiting for the train. She is looking for her iPod. It’s not in her coat. She suddenly sees that her purse is open. She can’t remember to have left it open. Suppose pickpockets are active in the train station.

**Conditional:** If Judy’s iPod is not in her purse, then someone has stolen it.

**Indicate how probable you find this conditional in the given context:**

Highly improbable  1  2  3  4  5  6  7  Highly probable
There was a significant difference between conditional probability and probability of conditional.

Thus, GSH does not hold.

Doesn’t this also cast doubt on SH? How can SH be descriptively correct (as earlier experimental data suggest) yet GSH be false?
Van Fraassen on GSH

Van Fraassen [1976]: if the semantics for “→” is relativized to epistemic states, then there do exist nontrivial probability functions satisfying (1), where “→” may be considered as logically well-behaved.

Lewis [1976]: if interpretation of the conditional varied with people’s belief states then that would preclude disagreements about conditionals.

Lewis’ objection presupposes that disagreeing parties must have exactly the same proposition in mind (otherwise there is no genuine disagreement).

That seems wrong: Hájek and Hall [1994], Douven and Dietz [2011].
Conclusion

Where do we stand?

- ‘Static’ part of epistemology:
  1. SH may be true after all: triviality results rely on a premise that is descriptively false;
  2. AT is false;
  3. the story about the acceptability / assertability of conditionals that we get in place of AT is messy!

- ‘Dynamic’ part of epistemology – updating on conditionals – is a mess too!