Titel / Title:  
Nontrivial Versions of Hume's Is-Ought Thesis and Their Logical Presuppositions

Autor / Author:  
Gerhard Schurz

TPD PREPRINTS  
Annual 2004 No.1  
Edited by Gerhard Schurz and Markus Werning

Vorveröffentlichungsreihe des Lehrstuhls für  
Theoretische Philosophie an der Universität Düsseldorf  
Prepublication Series of the Chair of  
Theoretical Philosophy at the University of Düsseldorf
Nontrivial Versions of Hume's Is-Ought Thesis and Their Logical Presuppositions

Gerhard Schurz, University of Düsseldorf (Germany)

1. Ought as Propositional Operator Expressing an Attribute of Propositions

A logical investigation of the Is-Ought problem is in a subtle position with respect to the question of ethical cognitivism. It has to presuppose at certain portion of cognitivism, but it has to remain ignorant with respect to rest of it. Let me therefore start my talk with a differentiation in the concept of cognitivism. What a logical investigation of the Is-Ought problem has to assume is what I call

(1.) Semantic Cognitivism: (a) Normative (ethical) assertions have semantical contents which can be represented as propositions, and (b) these propositions can be semantically evaluated by a meta-logical 'truth' or 'correctness' predicate.

With a proposition I mean a state of affairs expressed by a statement. So propositions are not syntactical but conceptual entities. With an ethical proposition, I always mean a normative or an evaluative proposition. I treat normative and evaluative propositions on par, and I think that Hume's thesis applies to both of them. But for sake of simplicity, I will focus on normative propositions and mention the generalization to value statements only in the margin.

Semantic cognitivism is weaker than the following position on which an investigation of Hume's these without prejudices must remain ignorant:

(2.) Epistemic Cognitivism: It is possible to justify the truth or correctness of normative (ethical) statements in an objective or subject-independent way.

If semantic cognitivism were false, then normative statements were not capable of
having a *logic*, and so, Hume's Is-Ought thesis would the true but completely trivial: normative statements can't be inferred from descriptive propositions simply because they can't be inferred at all. But if semantic cognitivism is true, then Hume's Is-Ought thesis is no longer trivial but becomes a matter of logical investigation.

That ethical propositions can be evaluated with respect to a metalogical truth or correctness predicate does, of course, *not* imply that they can be true in a realistic sense of truth. All what it implies is that it makes sense to speak of correct norms as opposed to incorrect ones, and indeed, this is already presupposed when a persons *asserts* a norm and *denies* its negation. This is all what is needed for a logic of ethical propositions. For logic is *not* about the nature of truth; logics is about the nature of *truth-preservation* – whatever truth substantially means: be it correspondence with the real world, or as correspondence with conceptual and ideal worlds, or whatever. The non-cognitivistic argument that ethical assertions cannot have a logic because they lack truth-values in the realistic sense is seriously mistaken – all necessary conditions for being capable of a logic are contained in conditions (a) and (b) of semantic cognitivism.

Thesis (a) of semantic cognitivism implies that ethical operators such as *being obligatory* or *being valuable* are *proposition-forming operators*. This means that they are unsaturated expressions which when applied to their argument placeholders yield a proposition. By thesis (b) of semantic cognitivism, the resulting propositions can be evaluated with respect to a truth or correctness predicate. So thesis (b) implies that these proposition-forming operators express *attributes* whose extension is the class of those argument placeholders which if the operator is applied yield a true or correct proposition. But what are the argument placeholders of ethical operators, concerning their grammatical type? Here comes my next crucial thesis:

(3.) The argument placeholders of ethical operators are not individuals, but propositions.
This is evident for normative operators – it does not make sense to say from an individual that it is obligatory or permitted. Only may only say that from an action, or from the result of an action, and more generally from a state of affairs. But the same goes for value operators. Even if you say from an individual that it is good – what you mean is that this individual is good insofar it causes certain states of affairs which are good. Moreover, if we assume the argument placeholders of normative operators to be individuals instead of propositions, then we cannot express natural analytic principles about Ought such as \( O(p \land q) \rightarrow Op \), because we cannot form conjunctions of individuals.

That the argument placeholders of ethical operators are propositions is expressed by saying that they are *propositional* operators. Semantically this implies that ethical operators express certain attributes of propositions (similar as the necessity operator, or the truth or falsity operator).

So far I have tried to clarify the semantical background of my enterprise. In the next section I try to explicate Hume's thesis.


First some terminological conventions. My logical framework will be the framework of a multimodal predicate language. Capital Arabic letters \( A, B, \ldots \) will range over arbitrary formulas of this language, \( F, G, \ldots, R, \ldots \) over predicates or relations; and I consider propositional variables \( p, q, \ldots \) as zero-placed predicates. I use \( \Box, \Diamond (\Box A :\leftrightarrow \neg \neg A) \) for alethic necessity and possibility, \( O, P, F \) for obligatory, permitted, forbidden \( (PA :\leftrightarrow \neg O \neg A; FA :\leftrightarrow O \neg A) \). Further propositional operators will, when I need them, be represented by underlined capital letters, such as \( \underline{V} \) for being valuable or \( \underline{B} \) for it is believed.

Hume’s thesis is usually formulated as the claim that one cannot infer *normative* statements from consistent *non-normative* statements. Thereby, normative statements are statements having non-trivial normative content, while non-normative statements
lack such content. But from a syntactic viewpoint, there is no such simple dichotomic division of statements into normatives and non-normative ones. We only have the following simple trichotomic division:

*Descriptive statements:* They don't contain any (active) occurrence of a normative operator. Examples: $p$, $p \rightarrow q$, $\forall x(Fx \rightarrow Gx)$. Thus, also alethic necessity statements count as descriptive.

*Purely normative statements:* All of their atomic subformulas lie in the scope of some (active) normative operator. Examples: $Op$, $Op \rightarrow Oq$, $\forall x(OFx \rightarrow PGx)$.

*Mixed statements:* Statements which are neither descriptive nor purely normative. Some of their atomic subformulas lie in the scope of some (active) normative operator, and some of them lie outside the scope of any (active) normative operator. Examples: $p \rightarrow Oq$, $\forall x (Fx \rightarrow PGx)$.

In what follows, D will always denote a purely descriptive statement, D will denote a set of purely descriptive statements, and N will denote a purely normative statement. The restriction to so-called active normative operators is only necessary if the language contains operators expressing subjective attitudes such as belief or desire. I call an occurrence of a normative operator active in a statement iff it does not lie in the scope of some subjective attitude operator; otherwise I call this operator occurrence inactive. Statements who's normative components lie in the scope of a belief operator are obviously descriptive, as in the following examples:

*Purely normative:* Possession of fire weapons is forbidden
*Descriptive:* Peter believes that possession of fire weapons is forbidden

Obviously, descriptive statements don't have any normative content, and purely normative statements which are not logically true do have normative content. But what about the normative content of mixed statements? Should we count disjunctive state-
ment such as \( p \lor Oq \) as having normative content, or not? Prima facie this is not clear. One may hope that a reasonable distinction among mixed statements between normatively contentful and normatively contentless ones can be found by more refined considerations. Prior's paradox of (1960), however, has completely destroyed this hope. His paradox shows that there simply is no reasonable way to divide the class of statements dichotomically into normative and non-normative ones, because every such dichotomy would force us to accept certain Is-Ought inferences – in short: IOIs – although we would intuitively not want to consider these inferences as genuine IOIs. Consider the following two examples:

\[
\begin{align*}
(I1) & \quad p & \text{There is war.} \\
& \quad p \lor Oq & \text{Either there is war or you should marry; equivalent with: } \neg p \rightarrow Oq \\
(I2) & \quad \neg p & \text{There is no war.} \\
& \quad p \lor Oq & \text{Either there is war or you should marry} \\
& \quad \text{or you should marry } Oq & \text{You should marry}
\end{align*}
\]

\( p \) and \( \neg p \) are purely descriptive and hence non-normative; \( Oq \) is purely normative and hence normative, but what is the status of the disjunction \( p \lor Oq \)? If we consider it as normative, then the left inference (I1) would count as an IOI, and if we consider it as non-normative, then the right inference (I2) would count as an IOI.

For logically oriented philosophers Prior's paradox was a somehow shocking result. Several philosophers, among them Prior himself, MacIntyre (1981, 57), and in the German speaking area Wolfgang Stegmüller (1973, 51), draw the conclusion that a general logical proof of Hume's Is-Ought thesis is impossible. Other philosophers such as Harrison (1972, 72), Kutschera (1977), and Stuhlmann-Laeisz (1983) concluded that Hume's Is-Ought thesis can only be proved in a restricted form in which mixed statements are excluded. I have called this thesis the special Hume thesis \( \text{SH} \), and it asserts the following:
Special Hume Thesis SH: No purely normative conclusion which is not already logically true is inferable from a consistent set of purely descriptive premises.

SH has an ethically important counterpart which I call the converse special Hume thesis, abbreviated as cSH, and which says that no descriptive conclusion which is not already logically true is inferable from a consistent set of purely normative premises. It have proved that SH and cSH are logically equivalent (by means of nonmodal 1st order logic; Schurz 1997, 74, proposition 7); so every multimodal logic which satisfies SH satisfies cSH, and vice versa.

The restriction of Hume's thesis to SH is a serious drawback because the maybe most important kind of ethical statements are not categorial but conditional norms, which are mixed statements of the form \((\forall x)(D \rightarrow N)\) where the expressions in brackets are optional (examples: "Everyone who has kids ought to take care of them", "If you drive a car you must have a drivers license", etc.) Therefore, other philosophers, including Charles Pigden (1989) and myself (1991, 1997), have tried to tackle the problem of the normative content of mixed statements. But how?

Intuitively we would count neither of both inferences as a genuine IOI. The left inference is not an IOI because here, the conclusion \(p \lor Oq\) is intuitively without non-trivial normative content. The right inference is not an IOI because here the premise \(p \lor Oq\) has intuitively normative content. But it is the same formula! What this shows is the following: whether a mixed disjunction has nontrivial normative content or not depends on the logical role which the statements plays in the given argument. But in what does this different logical role consist? Has it to do with the role of conclusion versus premise? This hope is quickly destroyed by another example of Prior where \(p \lor Oq\) figures as a conclusion like in example (I1) but now has non-trivial ethical content. Prior's example is full of irony, but in my mouth his example could sound somewhat politically incorrect, so I prefer to present you a different example:
So what is it which gives a mixed conclusion or premise in an argument an inferentially nontrivial normative content? An answer to this question is provided by the replacement criterion of inessentiality of subformulas. In the left inference, $p \lor Oq$ does not have relevant ethical content because the normative subformula $Oq$ in the conclusion of left inference is completely inessential, which means that it can be replaced by any other formula whatsoever, salva validate of the inference, i.e., without that this replacement makes the inference invalid. In the right inference, the disjunction $p \lor Oq$ has a relevant ethical content, because here the subformula $Oq$ of the second premise is essential; a replacement of it by an arbitrary new formula would make the inference invalid. Likewise, in the third inference the conclusion $p \lor Oq$ has relevant ethical content because here $Oq$ is an essential subformula, i.e., it cannot be replaced in an arbitrary manner without making the argument invalid.

This replacement criterion of inessentiality, or of irrelevance as I call it, has an interesting history. It has first been developed by Stephan Körner (1947) and J. P. Cleave (1973/74) who applied replacements to single subformula occurrences. Since 1986 the replacement criterion has been applied by Paul Weingartner and myself in Salzburg in order to solve paradoxes in philosophy of science and deontic logic (Weingartner/Schurz 1986, Schurz 1991b); and later on I applied the replacement criterion to the Is-Ought problem (1991a, 1997). However, I applied replacements not only to single but to multiple subformula and predicate occurrences. Prior and Pigden have applied the replacement criterion, too; so this criterion is well at home in New Zealand. However, Prior and Pigden apply replacements not to subformulas or predicates but directly to the normative operator. In (1960, 204) Prior had observed that in the conclusion of the first inference, the single occurrence of the obligation operator
is *contingently vacuous* in the sense that it can be replaced by any other sentential operator salva validities. In (1989, 133f) Charles Pigden has generalized Prior’s replacement to multiple operator replacements, similar as I did this in Salzburg with multiple predicate replacements. Pigden illustrated the necessity of multiple replacements at hand of another counterexample of Prior – namely the following:

(I4)

\( p \rightarrow q \) \hspace{1cm} If someone is an undertaker, then (s)he is a church officer.

\( (q \rightarrow \text{Or}) \rightarrow (p \rightarrow \text{Or}) \) \hspace{1cm} If church officers should do X, then undertakers should do X.

No single operator occurrence is replaceable here, salva validitate. Still, the conclusion of this inferences is normatively inessential, because O can be simultaneously replaced on both occurrences by any other sentential operator, salva validitate. Note that instead of replacing the operator, we may replace also the entire subformula Or, or just the propositional variable r, and get the same effect.

For the examples discussed so far the difference of replacing operators versus replacing predicates or subformulas does not seem to make a big deal, but there is a deep point lurking behind this difference – so deep that it forces us into debate about the nature of logic. For as Pigden (1989, 138) observes, the Prior-Pigden method of operator replacements breaks down as soon as we pass to standard systems of deontic modal logic. In such logics, the obligation operator does no longer play the role of a non-logical symbol who's interpretation is completely free. Rather, its interpretation is restricted in order to satisfy special logical or analytic axioms, such as the mentioned axiom of monotony, \((dM) \text{O}(A \land B) \rightarrow \text{OA}\). By conjoining this axioms to a descriptive conclusion, we can construct trivial Is-Ought-inferences of the following sort:
In these inferences, it is not possible to replace the obligation operator uniformly by any other sentence operator salva validitate, because not every sentence operator satisfies the principle of monotony. And the same holds for all other specific axioms of deontic logics. Nevertheless, the conclusion of these two inferences is normatively inessential, because the normative components of the conclusion have not really been derived from the descriptive premise $p$; rather, they have been introduced by conjoining to $p$ a logical truth of deontic logic. To this kind of normative inessentiality, the operator replacement criterion does not apply. However, my predicate replacement criterion still applies: every propositional variable of the conclusion can be uniformly replaced on all of its occurrences which lie in the scope of an obligation operator, salva validitate. After checking this replacement method at hand of various examples I was convinced that it was the right one. This led me to the following explication of the general Hume thesis (Schurz 1997, ch. 3.3):

**General Hume thesis GH:** Every mixed conclusion which is inferable from descriptive premises is *normatively irrelevant* in the following sense: every predicate of the conclusion can be replaced, salva validitate, by an arbitrary new predicate (of the same arity) on exactly those occurrences which lie in the scope of a normative operator.

The general Hume thesis has three ethically important counterpart versions. All of them are logically equivalent with GH by means of nonmodal 1st order logic; so GH holds in a multimodal logic iff any of its three counterpart versions holds in it (Schurz 1997, 89f, proposition 9). One of these counterpart version is of importance for Prior's example (I2), and it says: every premise set which contains a mixed premise
and which logically implies a purely normative conclusion is completely is-irrelevant in the following sense: every predicate of the premise set can be replaced, salva validitate, by an arbitrary new predicate (of the same arity) on exactly those occurrences which do not lie in the scope of a normative operator. In Prior's example (I2): "p, ¬p ∨ Oq / Oq", the first premise is descriptive, the second premise is mixed and the conclusion is purely normative. Of course, neither the premise set nor the conclusion is Ought-irrelevant in this example, because the propositional variable "q" is neither salva validitate replaceable in the O-scope of the second premise, nor is it replaceable in the O-scope of the conclusion. However, the premise set is is-irrelevant, because all occurrences of the propositional variable p outside of O-scopes are simultaneously replaceable in the premise set salva validitate. This means that although (I2) draws a normatively relevant conclusion from a mixed premise set, it nevertheless does not establish a relevant connection between descriptive proposition "p" and the normative proposition "Oq".

3. Hume's Is-Ought Thesis and the Question of 'Genuine' Logic

With this formulation of GH the way seems to be free for proving Hume's thesis for a large class of multimodal deontic logics including all standard systems and many more. Pigden, however, does not want to go this way. He prefers the New Zealand way of applying replacements to that what is directly at stake, namely to the obligation operator. But according to operator replacement criterion it seems that we are forced to accept the last two inferences and many more ones as genuine Is-Ought inferences. Prior (1960, 206) has indeed drawn this consequence. Pigden, however, does not draw this consequence. In order to avoid this consequence, he takes a big step: he refutes modal logic as a proper logic at all. According to Pigden even the axiom O(p △ q) → Op, in spite of its weakness, is not a proper logical principle (1989, 139-142). In my (1997), however, I investigate the validity of Hume's thesis in multimodal logics with arbitrary specific axioms. It seems that this constitutes a funda-
mental difference between us. Let me now try to explain why I think that this is not so. It is more a difference in division of labor.

At this point, I remind you to my discussion of Ought as a predicate versus an operator in the beginning of my talk. What Pigden has in mind is to conceive Ought just like an ordinary 1st order predicate. If this would be literally true, then we would obtain the general as well as the special Hume thesis as a trivial corollary of the theorem of uniform substitution for non-logical predicates in 1st order logic, which has first been proved by Kleene (1971, 155-162), as follows:

<table>
<thead>
<tr>
<th>Pigden's GH</th>
<th>Special Hume Thesis</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td>hence by</td>
<td>hence D must be inconsistent</td>
</tr>
<tr>
<td>unif. subst.</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>A[F/O]</td>
</tr>
<tr>
<td></td>
<td>Oa in 1st order logic (F an arbitarry predicate)</td>
</tr>
<tr>
<td>A</td>
<td>A[Op/O]</td>
</tr>
<tr>
<td></td>
<td>OA in operator logic (Op an arbitrary operator)</td>
</tr>
</tbody>
</table>

Pigden is well aware of how troublesome it is to represent actions or propositions by individual terms and ethical properties by ordinary predicates. He therefore formulates his general Hume thesis in the framework of Steven Kuhn's general operator logic (1981). In this logic, sentential operators are treated like non-logical predicates without any specific axioms and with a completely free interpretation. As a result, a uniform substitution theorem for operators holds in Kuhn's logic. However, the logical framework to which Pigden refers is not so different from my framework of multimodal logics. Both frameworks can be unified. Kuhn (1981, 496) remarks that his logic is equivalent with the general operator logic introduced by Cresswell (1972) as a certain weakening of classical multimodal logic with neighborhood semantics. Classical multimodal logic has been introduced by Segerberg (1971) as the minimal logic of arbitrary propositional operators, that is, of operators applying to propositions. They have a completely free interpretation, too, except that they must respect invariance with respect to logical equivalence transformations of their sentential ar-
arguments, and hence they have to satisfy the equivalence rule (E)

$$\text{(E) } \models A \leftrightarrow B \quad \text{and} \quad \models \text{Op}(A) \leftrightarrow \text{Op}(B) \quad \text{for every propositional operator Op}$$

In neighborhood semantics the interpretation of a propositional operator is just its extension in the ordinary sense, namely that set of propositions to which the operator truthfully applies. Thereby, propositions are represented as sets of possible worlds in which the proposition holds. It is easy to show that Pigden's generalized Hume thesis as well as the special Hume thesis can proved in the same way as above also for the minimal classical multimodal logic; and this logic is at the same time the minimum of the range of logics in which I have carried out my own investigations. The difference between neighborhood semantics and the Kuhn-Cresswell semantics for sentential operators is just that the latter has an additional set of nonclassical possible worlds in order to invalidate (E); but this difference is not important for the point which I make.

Let me summarize: Pigden has a established a basic and minimal version of a generalized Hume thesis based on operator replacements, and a special Hume thesis, which hold in logics in which operators have an arbitrary interpretation without any specific principles governing them. I come now to my point of division of labor: what I do is not in conflict with Pigden's version of Hume's thesis – I just want to do more. Because I have recognized that one can do more. Already Kutschera (1977) and Stuhlmann-Laeisz (1983) have shown that the SH holds for a certain class of standard alethic-deontic modal logics, and I have started my investigation by generalizing their result. Later I was able to show that the GH can not only be proved for the minimal classical logic, but a very large class of multimodal logics including all standard systems. This presupposes, of course, that the replacement criterion is applied to predicates and not to operators. Moreover, I try to push this enterprise to its limit – that is, I proved not only sufficient but also necessary conditions for Hume's thesis.

Why is this important? Well, all ethical theories assume special meaning principles for ethical operators from which they reason. If Hume's thesis would only hold if
any such special principles are excluded, then ethicists would be right in claiming that for their purposes Hume's thesis is not interesting. In fact, it is one of the central claims of Pigden that Hume's thesis is rather trivial, and I agree that this is true for his minimal version of Hume's thesis, but it is not true for my versions of Hume's thesis. I also think that in his famous passage (1739/40, 177f), Hume himself did not want to exclude special meaning principles about Ought; what he emphasized was only that Ought is entirely different from Is. My version of Hume's thesis does not exclude special analytic principles about Ought but applies to all kinds of ethical theories and argumentations. It helps to detect invalid ethical arguments as well as to find unreflected ethical premises. So I think that my version of the Hume's thesis is ethically highly non-trivial.

If someone objects that multimodal deontic logics are not genuine logics because Ought is not a genuinely logical symbol, then maybe my best reply is to ask: so what? But I want to say more on this point. Logic explicates the analytic meaning principles of logical symbols such as "not" or "and". Extralogical analytic meaning principles explicate the meaning of nonlogical symbols such as "bachelors are unmarried men". Is "Ought" more like "and" or more like "bachelor"? This is a rather controversial matter. I cannot go and I don't need to go into details here. Because I doubt that this question is it important for the Is-Ought question. It does not matter whether you consider Ought as a logical, non-logical or semi-logical symbol, and whether you consider special axioms for deontic systems a logical axioms or as extralogical analytical postulates. The point is that special meaning principles are used in all ethical systems, and that it is important to know whether or not they enable one to infer Ought from Is.

So, special principles of deontic logic need not be genuinely logical. But I am inclined to assume that, at least, they should express meaning principles; in other words, they should be analytically true. Even this may be doubted, because one may object with Quine that there does not exist a clear borderline between analytic and synthetic truths. Again, my best reply will be – so what? Even if deontic principles
are not genuinely analytic, they are assumed in ethical theories, and we want to know what is derivable with help of them. But observe there is still a clear difference between principles of deontic logics and material norms: the former are structural in the sense of being closed under substitution – not for operators, but for – for predicates and individual terms. Hence there are formulated with schematic letters A, B,... for which you can insert arbitrary sentences. Material norms, instead, are not closed under such substitutions. Here are some examples:

<table>
<thead>
<tr>
<th>Structural ethical principles</th>
<th>Material ethical norms</th>
</tr>
</thead>
<tbody>
<tr>
<td>(dD) OA → PA</td>
<td>(dD) O(earth remains ecologically intact)</td>
</tr>
<tr>
<td>(MO) A → OA</td>
<td>(a ‘BP’, see below)</td>
</tr>
<tr>
<td>∀ humans x,y: O(x respects y’s dignity)</td>
<td></td>
</tr>
</tbody>
</table>

Only structural principles are suitable as axioms of deontic logics. Therefore, what one usually calls a multimodal logic may also be called a structural multimodal theory. In what follows I will use these terms equivocally. Of course, not all structural principles are reasonable analytic meaning principles. On independent meta-ethical reasons I believe that the intersubjective justifiability of ethics has to be based on intersubjectively stable meaning principles for ethical expressions – but this is an independent question to which I will turn at the end of my talk. What I wanted to point here is that my versions of Hume's thesis are ethically non-trivial regardless of one's position concerning the logic-analytic-synthetic distinction.

To prove the general and the special Hume thesis for a broad class of multimodal deontic logics is also logically non-trivial – because of the following reason: the validity of Hume's thesis cannot be transferred from a given logical system \( L \) to a stronger logical system \( L^+ \), nor can it be transferred to a weaker logical system \( L^- \). This may sound surprisingly, but it has a simple reason: the general as well as the special Hume thesis involve a derivability condition and a non-derivability condition; the former cannot be transferred to weaker systems, and the latter cannot be transferred to stronger systems. Therefore, in order to pursue my goal, I had to develop
new syntactic and semantic proof-techniques which apply at once to very large classes of multimodal logics. This was the really hard logical task. Let me now turn the most important results of my logical investigation.

5. Hume's Thesis in Alethic-Deontic (Multimodal) Logics without Bridge Principles
Let me start with a list of well-known axioms and rules in the lattice of alethic-deontic logics ordered by logical strength, which is presented in fig. 1 on the next page. BP is an abbreviation for Is-Ought bridge-principles; in short bridge principles. BPs are not just arbitrary mixed axioms. They must connect Is and Ought in a relevant way according to the definition given in fig. 1. Multimodal logics which are axiomatizable without BPs can be considered as the natural Hume-limit, because in his famous passage, Hume excluded BPs as meaning principles. Let me turn now to my main theorems (cf. Schurz 1997; 92, theorem 1; 161, proposition 21.1; 102, theorem 2; 165, proposition 22.1):

**Syntactic GH-theorem (suff & necc):** GH holds in a alethic-deontic (multimodal) logic $L$ iff $L$ is axiomatizable without BPs.

**Semantic GH-theorem (suff & necc) – established for frame-complete regular logics:** GH holds in $L$ iff $L$ is characterized by all is-ought-separated frames for $L$.

SH can only be derived from GH if the alethic fragment of the underlying logic is Halldén-complete, which means that for all formulas $A$, $B$ having no predicates in common, the following holds: $A \lor B \in L$ iff $A \in L$ or $B \in L$. Most but not all standard systems of alethic modal logic are Halldén-complete. Therefore, the range of SH is more restricted than that of GH (cf. Schurz 1997, 118, theorem 1; 165, proposition 22.3; 120, proposition 16; 165, proposition 22.4):
Propositional and 1st Order Axioms and Rules

For $\vdash A \leftrightarrow B$ / $\vdash oA \leftrightarrow oB$ Classical $\rightarrow$ Neighborhood frames

For O: $o \in \{ , O\}$

Classical $\rightarrow$ Neighborhood frames

Monotonic $\rightarrow$ Kripke frames with queer worlds

(C) $oA \wedge oB \rightarrow o(A \wedge B)$ Regular $\rightarrow$ Kripke frames with queer worlds

(aV) $(p \lor \neg p)$

(鸠) $A \rightarrow A$ minimal -logic

(aT) $A \rightarrow A$ minimal -logic

(aB) $\Diamond A \rightarrow A$ Not for O! Only $(dT') o(OA \rightarrow A)$

(a4) $A \rightarrow A$ (d4) $OA \rightarrow OOA$

(a5) $\Diamond A \rightarrow \Diamond A$ strongest 'standard' (d5) $PA \rightarrow OPA$ strongest 'standard'

(minimal O-logic (eventually without dV) $\rightarrow$ Kripke frames

[optional: BF - constant domain; class./free quant. rules - rigid/nonrigid designators]

Normal $\rightarrow$ Kripke frames

Until here: combinations of mono-modal axioms Combined & unmixed logics

Here we may add bimodal axioms which are nevertheless purely normative:

For example: $OA \rightarrow OA$, $OA \rightarrow O A$ Unmixed logics

Here we may add mixed axioms which are nevertheless not BPs:

E.g.: $(A \rightarrow A) \lor (OB \rightarrow OOB)$ Logics without BPs = the natural Hume Limit

Here we may add BPs. E.g. $A \rightarrow OA$ (Must-Ought); $A \rightarrow OA$ (Is-Ought)

In multimodal logics: In-the-interest-of-all$(A) \rightarrow O(A)$ (weak Pareto principle)

Definition of BPs: they contain at least one schematic letter both inside a normative scope and outside of any normative scope.

Logics with BPs

increasing strength of logic

Fig. 1: The lattice of alethic-deontic logics
**Suff. SH-theorem** — established for frame-complete unmixed regular logics: If $L$'s alethic (descriptive) sublogic is Halldén-complete, then SH holds in $L$.

**Necc. SH-theorem** — established for frame-complete combined & unmixed regular logics: SH holds in $L$ only if $L$'s alethic (descriptive) sublogic is Halldén-complete.

In order to generalize these results to arbitrary multimodal logics containing subjective attitude operators such as belief or desire, I have proved that the above theorems generalize to premises in which every ethical operator occurring in them is inactive, that is, it lies in the scope of some subjective attitude operator. I have called the corresponding theorem the *Max Weber thesis* (Schurz 1997, 150, theorem 7; 154, theorem 8). Max Weber was the first philosopher of science who in his "Wissenschaftslehre" (1917, 499-501) argued that sociology can be a purely descriptive science, free from ethical valuations, because assertions about the factual acceptance of ethical norms or values in given societies are descriptive assertions.

The ethical use of GH can be illustrated by examples. Assume an ethicist claims to have derived from purely descriptive premises the following conclusions:

All beings capable of suffering have the right to live

All men should help their next, if they can.

Asked for his or her logic, the ethicist assures that he or she has only used standard axioms and rules which did in particular not contain bridge principles. Then we ask him or her whether he or she would also consider the following assertions as inferable from his or her premises.

For all actions $X$: All beings capable of suffering have the right to do $X$.

All men should do $X$ to their next, if they can help their next.

Certainly, the ethicist will vehemently deny that. Now we can point out to him or her that by our theorem about GH, this argument can impossibly be valid. He or she must rather have presupposed some hidden bridge principles in his or her argumentation or
theory.

One may be curious to know how the above irrelevant conclusions can ever be inferred from descriptive premises without BPs. The answer is simple: for example, the descriptive premises may imply the statement that no being is capable of suffering, and that nobody is able to help their next; then the conclusions would be a trivial irrelevant consequence. This illustrates once more why the formulation of the GH with help of a relevance criterion is necessary.

Turning now to the SH, the best known example of an alethic modal logic which is not Halldén-complete is the logic $\mathbf{aG}$ where $\mathbf{G}$ is interpreted as provability in arithmetics. This logic, which is axiomatized as a normal logic plus the axiom (G) $(A \rightarrow A) \rightarrow A$, validates the purely alethic inference $\neg \bot \vdash \neg \neg \bot$, which says the following: if arithmetic is consistent, then it cannot prove its own consistency (this is the 2nd version of Gödel's incompleteness theorem). By simple proof steps, one derives from there the Is-Ought inference $\diamond A \vdash OB$ for arbitrary $A$ and $B$, which violates SH. Fortunately, GH still holds and it tells us that all Is-Ought-inferences due to a lack of Halldén-completeness are normatively irrelevant. So although SH is violated, the spirit of Hume's thesis is still intact, which demonstrates once again the importance of GH. The realm of 'logics' where Hume's thesis may substantially fail is the realm of logics containing essential BPs. To these logics I turn now.

5. The Logical versus the Analytic (or Conceptual) Is-Ought Problem

My point of view implies a significant shift concerning the nature of the Is-Ought question. Logic alone can no longer give an absolute answer to it. Everything depends on what special structural meaning principles are assumed about Ought, and in particular, whether these principles contain bridge principles. Therefore I suggest that the Is-Ought problem divides into two problems, one which can be answered by logic alone, and another one which needs independent meta-ethical investigation:
The logical Is-Ought problem: Give sufficient and necessary conditions for validity of interesting versions of Hume's thesis in terms of general (syntactical or semantical) properties of the structural axioms of an underlying multimodal logic.

Hume's thesis ('logical' in nature): "Non-trivial" IOIs are not possible in a logic if (and only if) this logic is axiomatized without "non-trivial" Is-Ought-interaction principles.

The analytical (or conceptual) Is-Ought problem: Find out which structural principles for ethical operators are analytically plausible – in the sense of being plausible as meaning principles for Ought. In particular: are there any analytically plausible bridge principles?

Moore's thesis ('analytic' in nature): No bridge principle is analytically plausible.

On purpose I have Hume's thesis expressed in a vague way: by inserting different precise notions for "non-trivial" in these two places you obtain different versions of Hume's thesis. Although my investigations have answered the logical Is-Ought problem to a significant extent, various logical problems are still open and are waiting for an answer. For example, so far nobody has provided sufficient and necessary conditions for SH, and nobody has investigated SH in logics which are weaker than regular. In particular, nobody so far has investigated Hume's Is-Ought thesis in defeasible deontic logics.

I think that Hume's thesis was logical in nature. The analytical or conceptual Is-Ought-separation thesis was Moore's thesis, but not a thesis of Hume. Hume accepted certain bridge principles about which we are been informed in Annette Baier's paper in this volume. Thus, by separating the logical from the analytical Is-Ought problem we can see that Hume was not really incoherent. Observe that my distinction between the logical and the analytical Is-Ought problem is different from the traditional distinction between the question of derivability and the question of definability of Ought.
(Frankena 1939, 475f). Non-logical but analytically plausible bridge principles need not always be definitions; they can have any logical form. It is the question of derivability itself which splits into the logical Is-Ought problem – roughly, what is derivable from what – and the analytic Is-Ought problem – roughly, what are analytically plausible structural axioms?

6. Analytically Plausible BPs and the Practical Hume Thesis

The following two bridge principles have a high analytic plausibility:

(OC) OA → ◊A  (Ought–Can)

(ME) (A → B) → (OA → OB)  (necessary-Means–Ends)

In normal logics (ME) is equivalent with, and in regular logics (ME) is implied by:

(MO) A → OA  (Must–Ought)

Nevertheless, the three principles are not substantial Is-Ought-connection principles. Rather, they are functional principles in the use of ethical arguments. According to (OC), practical impossibilities restrict the space of possible fundamental norms. According to (ME), practical necessities make it possible to infer from one or just a few fundamental norms a multitude of derived norms. Both principles are of highest importance for all applied ethicists. What you can’t do with the two principles, however, is to derive non-trivial categorial or conditional norms or permissions from non-normative premises. Let me call a categorial or conditional norm or permission a practically normative statement – these statements have the following form (brackets are optional):

Categorial ( ) (∀x) (→)OD  Conditional: ( ) (∀x)( ) (D1 → (¬)OD2 )

Let P range over practically normative statements. I was able to prove is that all practically normative statements which are derivable from descriptive premises with help of these BPs are trivial in the following sense: already the factual truth of the proposi-
tion which is derived as being permitted or obligatory is logically implied by the same descriptive premises. I explicate this as follows (Schurz 1997, 132, theorem 6):

(1) An IOI $D \vdash^L P$ is called **practically trivial** iff also $D \vdash^{P^{-O}}$ is valid, where $P^{-O}$ results from $P$ by omitting the normative operator.

(2) The practical Hume thesis $PH$ holds in a logic $L$ iff every $L$- inference $D \vdash^L P$ is practically trivial.

**PH-Theorem** – established for combined & unmixed base logics plus BPs: If $L$'s BPs are among $\{MO, ME, OC\}$, then $PH$ holds in $L$.  

The usefulness of $PH$ can be illustrated at hand of examples. Naturalistic ethicists typically start from some necessary facts about human nature such as all men are born free, and ultimately arrive at practically normative conclusions about natural rights such as all men have the right to be free. They purport to have presupposed no other BP than MO, ME or OC. But of course, they wouldn't want to infer from their premises that men are free anyway, because then their proclamation of natural rights would not be interesting. By our theorem on $PH$ we can point out that this is impossible: they must have presupposed some hidden BP different from MO, ME or OC. Note that $PH$ only says that practically normative statements derived from descriptive premises are trivial; other kinds of inferences derived with help of these BPs can be highly non-trivial.

The $PH$-theorem is only a first step in the logical investigations of bridge principles. Some other results have been established by Galvan (1988), and there are several open problems in this field. Figure 2 below presents a graphical landscape of my logical results in the Is-Ought problem.
7. Outlook: In Search for Analytically Plausible and Practically Non-Trivial BPs

In my (1997, ch. 11, 12) I have devoted two chapters to this search by studying the fundamental principles of the most important types of ethical theories. Interestingly all of them, even the principles of autonomistic ethical theories, turned out to be bridge principles. My conclusion was that the question of justifiability of ethics depends strongly on the question of justifiability of analytically plausible BPs, and today I still think that this is true. Let mention some of these BPs (the universalization-BP is not contained in my 1997):

Some substantial BPs – the first three PH admit practically non-trivial IOIs:

(1) BP of full utilitaristic aggregation theories (Mill, etc.):
An action A is obligatory iff A is a necessary condition of all possible collective ac-
Diagnostics with maximal collective utility.

**Diagnosis:** not analytically plausible, because the explication of "maximal collective utility" is highly controversial.

(2) **BP of minimal aggregation theories – strong Pareto principle:**

Strong Pareto principle: An action is ethically valuable if it increases the utility for some persons ceteris paribus, that is, without diminishing the utility for other persons.

**Diagnosis:** maybe analytically plausible, and non-empty, but its content is very weak

(3) **BP of intuitionistic [empiricistic] ethics (Moore [Hume]):**

An proposition A is ethically valuable if every human under normal cognitive [observation] conditions would spontaneously judge [would observe] that A is ethically valuable

**Diagnoses:** maybe analytically plausible, but its antecedent is almost empty.

(4) **Universalization-BP (Justice-BP; Kant,Hare ...):**

If two persons x, y agree in all properties out of a given class DES of ethically relevant descriptive properties, then they agree in their rights and duties;

**Diagnosis:** analytically plausible, except for the specification of 'DES'. I conjecture that this BP does not admit practically non-trivial IOIs.

While in my Is-Ought book my conclusion concerning analytically plausible BPs was rather skeptical, I am nowadays more optimistic towards this question. Let me end with a final perspective. Since Ross (1930) has developed his concept of a prima facie norm it has often be suggested that conditional norms should not be understood in the strict sense, but in a defeasible or normic sense (cf. Horty 1994, Nute 1977). So far, no one has investigated Hume's Is-Ought problem in a defeasible deontic logics. I think that based on some well-established connections between strict and defeasible conditionals (Poole 1988, Schurz 1997) one may be able to transfer many of my results to defeasible logics, but so far this is an unproved conjecture. But what seems clear to me is that the prospects for establishing analytically plausible and contentful
BPs are much higher if we understand conditional norms not in the strict but in the defeasible sense. With this optimistic remark I want to conclude my paper.

References


