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22. Dual oppositions in lexical meaning

Abstract
Starting from well-known examples, a notion of duality is presented that overcomes the shortcomings of the traditional definition in terms of internal and external negation. Rather duality is defined as a logical relation in terms of equivalence and contradiction. Based on the definition, the notion of duality groups, or squares, is introduced along with examples from quantification, modality, aspectual modification and scalar predication (adjectives). The groups exhibit remarkable asymmetries as to the lexicalization of their four potential members. The lexical gaps become coherent if the members of duality groups are consistently assigned to four types, corresponding e.g. to *some*, *all*, *no*, and *not all*. Among these types, the first two are usually lexicalized, the third is only rarely and the fourth almost never. Using the example of the German *schon* ("already") group, scalar adjectives and standard quantifiers, the notion of phase quantification is introduced as a general pattern of second-order predication which subsumes quantifiers as well as aspectual particles and scalar adjectives. Four interrelated types of phase quantifiers form a duality group. According to elementary monotonicity criteria the four types rank on a scale of markedness that accounts for the lexical distribution within the duality groups.

1. Preliminaries

Duality of lexical expressions is a fundamental logical relation. However, unlike others such as antonymy it enjoys much less attention. Duality relates *all* and *some*, *must* and *can*, *possible* and *necessary*, *already* and *still*, *become* and *stay*. Implicitly, it is even involved in ordinary antonymy such as between *big* and *small*. Traditionally duality is defined in terms of inner and outer negation: two operators are dual iff the outer negation of the one is equivalent to the inner negation of the other; alternatively two operators are dual iff one is equivalent to the simultaneous inner and outer negation of the other. For example, *some* is equivalent to *not all not*. Duality, in fact, is a misnomer. Given the possibility of inner and outer negation, there are always four cases involved with duality: a given operator, its outer negation, its inner negation and its dual, i.e. inner plus outer negation. Gottschalk (1953) therefore proposed to replace the term *duality* by *quaternality*.

In this article, a couple of representative examples are introduced before we proceed to a formal definition of the relation of duality. The general definition of duality is not as trivial as it might appear at first sight. Inner and outer negations are not always available for dual operators at the syntactic level whence it is necessary to base the definition on a semantic notion of negation. Following the formal definition of duality, a closer look is taken at a variety of complete duality groups of four, their general structure and their relationship to the so-called Square of Oppositions of Aristotle's.

Duality groups of four exhibit striking asymmetries: out of the four possible cases, two are almost always lexicalized, while the third is occasionally and the fourth almost never. (If the latter two are not lexicalized they are expressed by using explicit negation with one of the other two cases.) Criteria will be offered for assigning the four members of a group to four types defined in terms of monotonicity and "tolerance".

A general conceptual format is described that allows the analysis of dual operators as instances of the general pattern of "phase quantification". This is a pattern of second-order predication; a phase quantifier predicates about a given first-order predication that there is, or is not, a transition on some scale between the predication being false and being true, i.e. a switch in truth-value. Four possibilities arise out of this setting: (i) there is a transition from false to true, (ii) there is no transition from false to true, (iii) there is a transition from true to false; (iv) there is no transition from true to false. These four possibilities of phase quantification form a duality group of four. It can be argued that all known duality groups semantically are instances of this general scheme.

1.1 First examples

1.1.1 Examples from logic

Probably the best-known cases of duality are the quantifiers in standard predicate logic, $\exists$ and $\forall$. The quantifiers are attached a variable and combined with a sentence (formula, proposition), to yield a quantified sentence.

(1) a. $\forall \mathbf{x}$ P for every x P

b. $\exists$x P for at least one x P

Duality of the two quantifiers is stated in the logical equivalences in (2):

(2) a. $\exists x\, P \equiv \neg \forall x \neg P$

b. $\forall x\, P \equiv \neg \exists x \neg P$

c. $\neg \exists x\, P \equiv \forall x \neg P$

d. $\neg \forall x\, P \equiv \exists x \neg P$

Duality can be paraphrased in terms of **external negation** and **internal negation** (cf. article 63 *Negation*)**.** External negation is the negation of the whole statement, as on the left formula in (2c,d) and on the right formula in (2a,b). Internal negation concerns the part of the formula following the quantifier, i..e. the "scope" of the quantifier (cf. article 62 *Scope and binding*). For example, according to (2c) the external negation of existential quantification is logically equivalent to the internal negation of universal quantification, and vice versa in (2d). If both sides in (2c) and (2d) are negated and double negation is eliminated, one obtains (2a) and (2b), respectively. In fact the four equivalences in (2) are mutually equivalent: they all state that universal and existential quantification are duals.

Dual operators are not necessarily operators on sentences. It is only required that at least one of their operands can undergo negation ("internal negation"), and that the result of combining the operator with its operand(s) can be negated, too ("external negation").

Another case of duality is constituted by conjunction $\wedge$ and disjunction $\vee$; duality of the two connectives is expressed by De Morgan's Laws, for example:

(3) $\neg(A \wedge B) \equiv (\neg A \vee \neg B)$

These dual operators are two-place connectives, operating on two sentences, and internal negation is applied to both operands.

1.1.2 First examples from natural language

The duality relationship between $\exists$ and $\forall$ is analogously found with their natural language equivalents *some* and *every*. Note that sentences with *some* NPs as subject are properly negated by replacing *some* with *no* (cf. Löbner 2000: §1 for the proper negation of English sentences):

(4)	a.	***some*** *tomatoes are green*	≡	***not every*** *tomato is* ***not*** *green*
	b.	***every*** *tomato is green*	≡	***no*** *tomato is* ***not*** *green*
	c.	***no*** *tomato is green*	≡	***every*** *tomato is* ***not*** *green*
	d.	***not every*** *tomato is green*	≡	***som****e tomatoes are* ***not*** *green*

The operand of the quantificational subject NP is its 'nuclear' scope, the VP.

Modal verbs are another field where duality relations are of central importance. Modal verbs combine with infinitives. A duality equivalence for epistemic *must* and *can* is stated in (5):

(5)	*he* ***must*** *have lied*	≡	*he* ***cannot*** *have told the truth*

Aspectual particles such as *already* and *still* are among the most thoroughly studied cases of dual operators. Their duality can be demonstrated by pairs of questions and negative answers as in (6). Let us assume that *on* and *off* are logically complementary, i.e. equivalent to the negations of each other:

(6) a. *Is the light* ***already*** *on? – No, the light is* ***still*** *off.*

b. *Is the light* ***still*** *on? – No, the light is* ***already*** *off.*

1.2 Towards a general notion of duality

The relationship of duality is based on logical equivalence. Duality therefore constitutes a logical relation. In Model-theoretic semantics (cf. article 33 *Model-theoretic semantics*), meaning is equated with truth conditions; therefore logical relations are considered sense relations (cf. article 21 *Sense relations*). However, in richer accounts of meaning that assume a conceptual basis for meanings, logical equivalence does not necessarily amount to equal meanings (cf. Löbner 2002, 2003 § 4.6, 10.5). It could therefore not be inferred from equivalences such as in (4) to (6) that the meanings of the pairs of dual expressions match in a particular way. All one can say is that their meanings are such that they result in these equivalences.

In addition, expressions which exhibit duality relations are rather abstract in meaning and, as a rule, can all be used in various different constructions and meanings. In general, the duality relationship only obtains when the two expressions are used in particular constructions and/or in particular meanings. For example, the dual of German *schon* "already" is *noch* "still" in cases like the one in (6), but in other uses the dual of *schon* is *erst* (temporal "only"); *noch* on the other hand has uses where it does not possess a dual altogether (cf. Löbner 1989 for a detailed discussion).

The duality relation crucially involves external negation of the whole complex of operator with operands, and internal operand negation. The duality relationship may concern one operand (out of possibly more) as in the case of the quantifiers, modal verbs or aspectual particles, or more than one (witness conjunction and disjunction). In order to permit internal negation, the operands have to be of sentence type or else of some type of predicate expressions. For the need of external negation, the result of combining dual operators with their operands must itself be eligible for negation.

A first definition of duality, in accordance with semantic tradition, would be:

(7) Let q and q' be two operators that fulfil the following conditions:

  (a) they can be applied to the same domain of operands
  (b) the operands can be negated (internal negation)
  (c) the results of applying the operators to appropriate operands can be negated (external negation)

  Then q and q' are **duals** iff external negation of one is equivalent to internal negation of the other.

This definition, however, is in need of modification. First, "negation" must not be taken in a syntactic sense as it usually is. If it were, English *already* and *still* would not be candidates for duality, as they allow neither external nor internal syntactic negation. This is shown in Löbner (1999: 89f) for internal negation; as to external negation, *already* and *still* can only be negated by replacing them with *not yet* and *no more/not anymore*, respectively. The term 'negation' in (7) has therefore to be replaced by a proper logical notion.

A second inadequacy is hidden in the apparently harmless condition (a): dual operators need not be defined for the same domain of operands. For example, *already* and *still* have different domains: *already* presupposes that the state expressed did not obtain before, while *still* presupposes that it may not obtain later. Therefore, (8a) and (8b) are semantically odd if we assume that one cannot be not young before being young, or not old after being old:

(8) a. *she's already young*
  b. *she's still old*

These inadequacies of the traditional definition will be taken care off below.

1.3 Predicates, equivalence and negation

1.3.1 Predicates and predicate expressions

For proper semantic considerations, it is very important to carefully distinguish between the levels of expression and of meaning, respectively. Unfortunately there is a terminological tradition that conflates these two levels when talking of "predicates", "arguments", "operators", "operands", "quantifiers" etc.: these terms are very often used both for certain types of expressions and for their meanings. In order to avoid this type of confusion, the following terminological distinctions will be observed in this article: A "predicate" is a meaning; what a meaning is depends on semantic theory (cf. article 1 *Meaning in linguistics*). In a model-theoretic approach, a predicate would be a function that assigns truth values to one or more arguments; in a cognitive approach, a predicate can be considered a concept that assigns truth values to arguments. For example, the meaning of *has lied* would be a predicate (function or concept) which in a given context (or possible world) assigns the truth value TRUE to everyone who has lied and FALSE to those who told the truth. Expressions, lexical or complex, with predicate meanings will be called **predicate expressions**. **Arguments** which a predicate is applied to are neither expressions nor meanings; they are objects in the world (or universe of discourse); such objects may or may not be denoted by linguistic expressions; if they are, let us call these expressions **argument terms**. (For a more comprehensive discussion of these distinctions see Löbner 2002, 2003: §6.2) Sometimes, arguments of predicate expressions are not explicitly specified by means of an argument term. For example, sentences are usually considered as predicating about a time argument, the time of reference (cf. article 57 *Tense*), but very often, the time of reference is not specified by an explicit expression such as *yesterday*. The terms **operator**, **operand** and **quantifier** will all be used for certain types of expressions.

If the traditional definition of duality given in (7) is inadequate, it is basically because it attempts to define duality at the level of expressions. Rather it has to be defined at the level of meanings because it is a logical relation and logical relations between expressions originate from their meanings.

1.3.2 The operands of dual operators

The first prerequisite for an adequate definition of duality is a precise semantic characterization of the operands of dual operators ("d-operators", for short). Since the operands must be eligible for negation, their meanings have to be predicates, i.e. something that assigns a truth-value to arguments. Negation ultimately operates on truth-values; its effect on a predicate is the conversion of the truth value assigned. Predicate expressions range from lexical expressions such as verbs, nouns and adjectives to complex expressions like VPs, NPs, APs or whole sentences. In (4), the dual operators are the subject NPs *some tomatoes* and *every tomato*; their operands are the VPs *is/are green* and their respective negations *is/are not green*; they express predications about the tomatoes referred to. In (5), the operands of the dual modal verbs are the infinitives *have lied* and *have told the truth*; they express predications about the referent of the subject NP *he*; in (6), the operands of *already* and *still* are the remainders of the sentence: *the light is on/off*; in this case, these sentences are taken as predicates about the reference time implicitly referred to.

Predicates are never universally applicable, but only in a specific **domain** of cases. For a predicate P, its domain D(P) is the set of those tuples of arguments the predicate assigns a truth value to. The notion of domain carries over to predicate expressions: their domain is the domain of the predicate that constitutes their meaning.

In the following, **presuppositions** (cf. article 91 *Presupposition*) of a sentence or other predicate expression are understood as conditions that simply restrict the domain of the predicate that is its meaning. For example, the sentence *Is the light already on* in (6) presupposes (among other conditions) (p1) that there is a uniquely determined referent of the NP *the light* (cf. article 41 *Definiteness and indefiniteness*) and (p2) that this light was not on before. The predication expressed by the sentence about the time of reference is thus restricted to those times when (p1) and (p2) are fulfilled, i.e. those times where there is a unique light which was not on before. In general, a predicate expression p will yield a truth-value for a given tuple of arguments if and only if the presuppositions of p are fulfilled. This classical Fregean view of presuppositions is adequate here, as we are dealing with the logical level exclusively. It follows from this notion of presupposition that predicate expressions which are defined for the same domain of arguments necessarily carry identical presuppositions. In particular that is the case if two predicate expressions are logically equivalent:

Definition 1: logical equivalence
Let p and p' be predicate expressions with identical domains. p and p' are **logically equivalent** – **p ≡ p'** – iff for every argument tuple in their common domain, p and p' yield identical truth values.

1.3.3 The negation relation

The crucial relation of logical contradiction can be defined analogously. It will be called 'neg', the 'neg(ation) relation'. Expressions in this relationship, too, have identical presuppositions.

Definition 2: negation relation
Let p and p' be predicate expressions with identical domains. p and p' are **neg-opposites**, or **negatives**, of each other – **p neg p'** – iff for every argument tuple out of their common domain, p and p' yield opposite truth values.

Note that this is a semantic definition of negation, as it is defined in terms of predicates, i.e. at the level of meaning. The tests of duality require the construction of pairs of neg-opposites, or **neg-pairs** for short. This means to come up with means of negation at the level of expressions, i.e. with lexical or grammatical means of converting the meanings of predicate expressions.

For sentences, an obvious way of constructing a negative is the application (or de-application) of grammatical negation ('g-negation', in the following). In English, g-negation takes up different forms depending on the structure of the sentence (cf. Löbner 2000: §1 for more detail). The normal form is g-negation by VP negation, with *do* auxiliarization in the case of non-auxiliary verbs. If the VP is within the scope of a higher-order operator such as a focus particle or a quantifying expression, either the higher-order operator is subject to g-negation or it is substituted by its neg-opposite. Such higher-order operators include many instances of duality. English *all*, *every*, *always*, *everywhere*, *can* and others can be directly negated, while *some*, *sometimes*, *somewhere*, *must*, *always*, *still* etc. are replaced for g-negation by *no*, *never*, *nowhere*, *need not*, *not yet* and *no more*, respectively.

For the construction of neg-pairs of predicate expressions other than sentences, sometimes g-negation can be used, e.g. for VPs. In other cases, lexical inversion may be available, i.e. the replacement of a predicate expression by a lexical neg-opposite such as *on/off*, *to leave/to stay*, *member/non-member*. Lexical inversion is not a systematic means of constructing neg-pairs because it is contingent on what the lexicon provides. But it is a valuable instrument for duality tests.

1.4 Second-order predicates and subnegation

1.4.1 D-operators

D-operators must be eligible to negation and therefore predicate expressions themselves; as we saw, at least one of their arguments, their operand(s) must, again, be a predicate expression. For example, the auxiliary *must* in (5) is a predicate expression that takes another predicate expression *have lied* as its operand. In this sense, the d-operators are second-order predicate expressions, i.e. predicate expressions that predicate about predicates. D-operators may have additional predicate or non-predicate arguments. For an operator q and its operand p let 'q(p)' denote the morpho-syntactic combination of q and p, whatever its form. In terms of the types of Formal Semantics, the simplest types of d-operators would be (t, t) (sentential operators) and $((\alpha, t), t)$ (quantifiers); a frequent type, represented by focus particles, is $(((\alpha, t), \alpha), t)$ (cf. article 34 *Model-theoretic semantics* for logical types).

1.4.2 Negation and subnegation

When the definition of the neg-relation is applied to d-operators, it captures external negation. The case of internal negation is taken care of by the 'subneg(ation) relation'. Two operators are subneg-opposites if, loosely speaking, they yield the same truth values for neg-opposite operands.

Definition 3: subnegation opposites
Let q and q' be operators with a predicate type argument. Let the predicate domains of q and q' be such that q yields a truth value for a predicate expression p iff q' yields a truth value for the neg-opposites of p.

q and q' are **subneg(ation) opposites**, or **subnegatives**, of each other – **q subneg q'** – iff

\> for any predicate expressions p and p' eligible as operands of q and q', respectively: if p neg p' then q(p) ≡ q'(p').

(For operators with more than one predicate argument, such as conjunction and disjunction, the definition would have to be modified in an obvious way.)

If two d-operators are subnegatives, their domains need not be identical. The definition only requires that if q is defined for p, any subnegative q' is defined for the negatives of p. If the predicate domain of q contains negatives for every predicate it contains, then q and q' have the same domain. Such are the domains of the logical quantifiers, but that does not hold for pairs of operators with different presuppositions, e.g. *already* and *still* (cf. §5.1).

An example of subnegatives is the pair *always*/*never*:

(9) *Max* ***always*** *is late* ≡ *Max* ***never*** *is on time*

To be late and *to be on time* are neg-opposites, here in the scope of *always* and *never*, respectively. The two quantificational adverbials have the same domain, whence the domain condition in Definition 3 is fulfilled.

2. Duality

2.1 General definition

Definition 3 above paths the way for the proper definition of duality:

Definition 4: dual opposites
Let q and q' be operators with a predicate type argument. Let the predicate domains of q and q' be such that q yields a truth value for a predicate expression p iff q' yields a truth value for the neg-opposites of p.

q and q' are **dual (opposite)s** of each other – **q dual q'** – iff:

\> for any predicate expressions p and p' eligible as operands of q and q', respectively: if p neg p' then q(p) neg q'(p').

The problem with condition (a) in the traditional definition in (7) is taken care of by the domain condition here; and any mention of grammatical negation is replaced by relating to the logical relation neg. If q and q' and an operand p can all be subjected to g-negation NEG, duality of q and q' amounts to the equivalence of NEGq(p) and q'(NEGp).

2.2 Duality groups

Any case of duality involves, in fact, not only two dual expressions, but also the negatives and subnegatives of the dual operators. In total, these are four cases, not more. First, a dual of a dual is equivalent to the operator itself; the analogue holds for negation and subnegation. Therefore, if q is a d-operator and N, S, D are any morpho-syntactic operations to the effect of creating a negative, subnegative or dual of q, respectively, we observe:

(10) a. NNq ≡ q

b. SSq ≡ q

c. DDq ≡ q

Furthermore, the joint application of any two operations N, S or D amounts to the third:

(11) a. NSq ≡ SNq ≡ Dq

b. NDq ≡ DNq ≡ Sq

c. DSq ≡ SDq ≡ Nq

For the logical quantifiers, these laws can be read off the equivalences in (3); as for natural language, consider the following illustration for *already* and *still*. Let p be *the light is on*; and let us accept that *the light is off* is its negative, Np. Let q be *already*. q(p) is thus (12a). The subnegation of q(p), Sq(p), is gained by replacing p with Np in (12b). The negation of *already* is expressed by replacing it with *not yet* (12c). The dual of *already* is *still* (12d).

(12) a.	q(p)		*the light is already on*	=	*already*(*the light is on*)
b.	Sq(p)	= q(Np) =	*the light is already off*	=	*already*(*the light is off*)
c.	Nq(p)		*the light is not yet on*	=	*not yet*(*the light is on*)
d.	Dq(p)		*the light is still on*	=	*still*(*the light is on*)

The combination of S and N (the order does not matter) yields (13a), the application of Nq to Np; this is obviously equivalent to Dq. NDq(p) would be the negation of the dual of *already*(p), i.e. the negation of *the light is still on*; this is accomplished by replacing *still* with *not anymore*: *the light isn't on anymore*, which in turn is equivalent to *the light is already off*, i.e. Sq (13b). Finally, the combination of dual and subnegation is yielded by replacing *already* by its dual *still* and p by its negative. This is equivalent to applying the negative of *already*, i.e. *not yet* to p (13c):

(13) a.	NSq(p)	= Nq(Np) =	*the light is not yet off*	≡	*the light is still on*
b.	NDq(p)	= N *still* p =	*the light isn't on anymore*	≡	*the light is already off*
c.	DSq(p)	= *still* Np =	*the light is still off*	≡	*the light not yet on*

Due to the equivalences in (10) and (11), with any d-operator q, the operations N, S and D yield a group of exactly four cases: q, Nq, Sq, Dq – provided Nq, Sq and Dq each differ from q (see §2.3 for reduced groups of two members). Each of the four cases may be expressible in different, logically equivalent ways. Thus what is called a "case" here, is basically a set of logically equivalent expressions. According to (10) and (11), any further application of the operations N, S and D just yields one of these four cases: the group is closed under these operations. Within such a group, no element is logically privileged: instead of defining the group in terms of q and its correlates Nq, Sq and Dq, we might, for example, just as well start from Sq and take its correlates NSq = Dq, SSq = q, and DSq = Nq.

Definition 5: duality group
A **duality group** is a group of up to four operators in the mutual relations neg, subneg and dual that contains at least a pair of dual operators.

Duality groups can be represented as a square of the structure depicted in Figure 1.

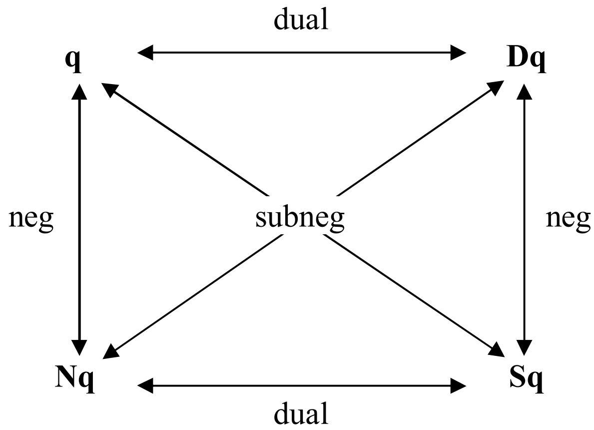

Figure 1: Duality square

Although the underlying groups of four operators related by neg, subneg and dual are perfectly symmetrical, duality groups in natural, and even in formal, languages are almost always deficient, in that not all operators are lexicalized. This issue will be taken up in §§3, 4 and 5 below.

2.3 Reduced duality groups and self-duality

For the sake of completeness, we will briefly mention cases of reduced (not deficient) duality groups. The duality square may collapse into a constellation of two, or even one case, if the operations N, D, or S are of no effect on the truth conditions, i.e. if Nq, Dq or Sq are equivalent to q itself.

Neutralization of N contradicts the Law of Contradiction: if q ≡ Nq, and q were true or false for any predicate operand p, it would at the same time be false. Therefore, the domain of q must be empty. This might occur if q is an expression with contradictory presuppositions, whence the operator is never put to work. For such an “idle” operator, negatives, subnegatives and duals are necessarily idle, too. Hence the whole duality group collapses into one case.

Neutralisation of S results in equivalence of N and D, since in this case Nq ≡ NSq ≡ Dq. One such example is the quantifier *some, but not all*: if a predication p is true in some but not all cases, its negation, too, is true in some, but not all cases. Quantificational expressions, nominal or adverbial, meaning “exactly half of” represent another case: if a predication is true of exactly half of its domain, its opposite is true of the other half: the quantification itself logically amounts to the subnegation of the quantification.

(14) a. q *half of the students failed*

b. Sq *half of the students didn’t fail*

When S is neutralized, the duality square melts down to q/Sq neg/dual Nq/Dq.

More interesting is the case of D neutralisation. If Dq ≡ q, q is its own dual, i.e. **self-dual**. For self-dual operators, N and S are equivalent. The domain of self-dual operators is generally closed under neg: since Nq ≡ Sq, q is defined for p iff it is defined for Np. The square reduces to q/Dq neg/subneg Nq/Sq. the phenomenon of self-duality encompasses a heterogeneous set of examples.

Polarity. The simplest example is g-negation NEG: N NEG(p) ≡ NNp ≡ p and S NEG(p) ≡ NEG(Np) ≡ NNp ≡ p. Similarly, if there were a means of expressing just positive polarity, say POS, this would be self-dual, too, since POS(Np) ≡ Np ≡ N POS(p).

Argument insertion. Let D be a domain of a first-order predicate p and u an element of D. Let I_u be the operation of supplying p with u as its argument. Then I_u applied to p yields the truth value that p yields for u: $I_u(p) = p(u)$. If we apply I_u to a neg-opposite of p (subnegation), we obtain the opposite truth value, and so we do if we negate the application of I_u to p (negation). I_u is exerted, for example, when a definite NPs is combined as an argument term with a predicate expression. This point is of importance for the discussion of NP semantics. It can be argued that all definite NPs are essentially individual terms; when combined with a predicate expression, they have the effect of I_u for their referent u. See Löbner (2000: §2) for an extensive discussion along this line. It is for that reason that (15b) and (15c) are logically equivalent:

(15) p "is on", p' "is off", u "the light"

a.	$I_u(p)$	=	$p(u)$	*the light is on*
b.	$SI_u(p)$	=	$I_u(p')$	*the light is off*
c.	$NI_u(p)$	=	$I_u'(p)$	*the light is not on*

"More than half". If the scale of quantification is discrete, and the total number of cases is odd, "more than half" is self-dual, too. Under these circumstances, external negation "not more than half" amounts to "less than half".

(16)

a.	q	*more than half of the seven dwarfs carry a shovel*
b.	Nq	*not more than half of the seven dwarfs carry a shovel*
c.	Sq	*more than half of the seven dwarfs don't carry a shovel*

Neg-raising verbs. So-called neg-raising verbs (NR verbs), such as "want", "believe", "hope", can be used with N to express S, as in (17). If this is not regarded as a displacement of negation, as the term 'neg-raising' suggests, but in fact as resulting from equivalence of N and S for these verbs, neg-raising is tantamount to self-duality:

(17) I don't want you to leave ≡ I want you not to leave

The question as to which verbs are candidates for the neg-raising phenomenon is, as far as I know, not finally settled (see Horn (1978) for a comprehensive discussion, also Horn (1989: §5.2)). The fact that NR verbs are self-dual allows, however, a certain general characterization. The condition of self-duality has the consequence that if v is true for p, it is false for Np, and if v is false for p, it is true for Np. Thus NR verbs express propositional attitudes that in their domain "make up their mind" for any proposition as to whether the attitude holds for the proposition or its negation. For example, NR *want* applies only to such propositions the subject has either a positive or negative preference for. The claim of self-duality is tantamount to the presupposition that this holds for any possible operand. Thereby the domains of NR verbs are restricted to such pairs of p and Np to which the attitude either positively or negatively obtains.

Among the modal verbs, those meaning "want to" like German *wollen* do not partake in the duality groups listed below. I propose that these modal verbs are NR verbs, whence their duality groups collapse to a group of two [q/Dq, Nq/Sq], e.g. [*wollen*, *wollen NEG*].

The generic operator. In mainstream accounts of characterizing (i-generic) sentences (cf. Krifka et al. 1995, also article 47 *Genericity*) it is commonly assumed that their meanings involve a covert genericity operator GEN. For example, *men are dumb* would be analyzed as (18a):

(18) a. GEN[x](x is a man; x is dumb]

According to this analysis, the meaning of the negative sentence *men are not dumb* would yield the GEN analysis (18b), with the negation within the scope of GEN:

(18) b. GEN[x](x is a man; ¬ x is dumb]

The sentence *men are not dumb* is the regular grammatical negation of *men are dumb*, whence it should also be analysed as (18c), i.e. as external negation w.r.t. to GEN:

(18) c. ¬ GEN[x](x is a man; x is dumb]

It follows immediately that GEN is self-dual. This fact has not been recognized in the literature on generics. In fact, it invalidates all available accounts of the semantics of GEN which all agree in analyzing GEN as some variant of universal quantification. Universal quantification, however, is not self-dual, as internal and external negation clearly yield different results (see Löbner 2000: § 4.2) for elaborate discussion).

Homogeneous quantification. Ordinary restricted quantification may happen to be self-dual if the predicate quantified yields the same truth-value for all elements of the domain of quantification, i.e. if it is either true of all cases or false of all cases. (As a special case, this condition obtains if the domain of quantification contains just one element u; both ∃ and ∀ are then equivalent to self-dual I_u.) The "homogeneous quantifier" ∃∀ (cf. Löbner 1989:179, and Löbner 1990: 27ff for more discussion) is a two-place operator which takes one formula for the restriction and a second formula for the predication quantified. It will be used in § 5.2.

Definition 6: homogeneous quantification
For arbitrary predicate logic sentences b and p,
$\exists\forall x(b : p) =_{df} \exists x(b \wedge p)$ if $[\exists x(b \wedge p)] = [\forall x(b \rightarrow p)]$, otherwise undefined.

The colon in '∃∀x(b : p)' cannot be replaced by any logical connective; [A] represents the truth value of A. According to the definition, $\exists\forall x(b : p)$ presupposes that $\exists x(b \wedge p)$ and $\forall x(b \rightarrow p)$ are either both true or both false. The presupposition makes sure that the truth of p in the domain defined by b is an all-or-nothing-matter: if p is true for at least one "b", it is true for all, and if it is false for at least on "b", it is false for all, whence it cannot be true for some. $\exists\forall x(b : p)$ can be read essentially as "the b's are p". (See Löbner 2000: §2.6) for the all-or-nothing-character of distributive predications with definite plural arguments.) $\exists\forall x(b : p)$ is self-dual: the b's are not-p iff the b's are not p. (A simple proof is given in Löbner 1990: 207f).

(19) $\neg\exists\forall x(b : p) \equiv \exists\forall x(b : \neg p)$

As a general trait of self-dual operators we may fix the following. Applying to a domain of predicates that necessarily is closed under negation, they cut the domain into two symmetric halves of mutually negative predicates: for every neg-pair of operands, they "make up their mind", i.e. they are true of one member of the pair and false of the other.

3. Examples of duality groups

3.1 Duality tests

The first thing to observe when testing for duality relations is the fact that they are highly sensitive to constructions in which a d-operator can be used. Strictly speaking, duality relations are defined for more or less specific constructions. For examples, there are different duality groups for German *schon* with stative IP focus, with focus on a scalar, time-dependent predicate, with focus on a temporal frame adverbial and others (Löbner 1989). Similarly, the duality groupings of modal verbs differ for epistemic vs. deontic uses. Many operators belong to duality groups only in certain constructions, but not when used in others. For example, German *werden* ("become") is the dual of *bleiben* ("stay") in the copula uses. As was pointed out by Schlücker (2008), there are, however, several uses of *werden* where it is not the dual match of *bleiben*, often because *bleiben* cannot even be used in certain constructions for *werden*.

For a given construction, the duality test involves the use of subneg-opposites for the operands and of neg-opposites for the whole. Often, even if available, g-negation is a problematic tool due to potential scope ambiguities and ambivalence between neg and subneg readings. For example, VP negation in sentences with universal quantifier subjects has ambiguous scope, unlike, of course, scopeless lexical inversion:

(20) *every light wasn't on*
subneg reading: ≡ *every light was off* (lexical inversion *on* vs. *off*)
neg reading: ≡ *not every light was on*

Similarly, for modal verbs g-negation sometimes yields a neg reading, sometimes a subneg reading:

(21) a. *she may not stay* (epistemic use) ≡ *she may leave*

b. *she may not stay* (deontic use) ≡ *she must leave*, <u>not</u> *she may leave*

Apart from these problems, g-negation may not be available, either for the operands or for the operators. For forming subnegatives it is generally recommended to use lexical inversion. Although not generally available, there are usually some cases of lexical neg-opposites in the domain of the operator which can be employed for tests. Since the operators can be assumed to operate in a logically uniform way on their operands, the findings on such cases can be generalized to the whole domain.

If g-negation is not available at the d-operator level, pairs of questions and negative answers can be used. The negative answer has to be carefully formed as to exactly match the question. This is secured if any denial of the question entails that answer, i.e. if that answer is the weakest denial possible. As mentioned above, *already* and *noch* are operators that bar both internal and external g-negation. The duality relations can be proved by using lexical inversion for the assessment of subneg (22), and the negative-answer test plus lexical inversion for duality (23).

(22) a. *the lights are **already** off* ≡ *the lights are **not** on **anymore***

b. *the lights are **still** off* ≡ *the lights are **not yet** on*

(23) a. *Are the lights **already** on? – No.* ≡ *the lights are **still** off*

b. *Are the lights **still** on? – No.* ≡ *the lights are **already** off*

3.2 Duality groups

3.2.1 Quantifiers

One group of instances of dual operators is constituted by various expressions of quantification. Different sets of quantifiers are used for quantifying over individuals, portions, times, places and other types of cases. Assessing the duality relationships within the respective groups involves the distinction between count and mass reference, collective and distributive predication and generic or particular quantification (cf: Löbner 2000: §3, §4 for these distinctions). The groups include nominal and adverbial quantifiers.
In the following, duality groups will be represented in the form '[operator, dual, negative, subnegative]' with non-lexical members in parentheses, '–' indicates a case that cannot be expressed. Throughout, the existential quantifier is chosen as the first member of each group. The additions 'pl' and 'sg' indicate the use with plural or singular, respectively. Table 1 displays a collection of duality groups of logical operators.

type of quantification	q	dual	negative	subnegative
particular or generic nominal distributive q.	*some* pl	*every* sg	*no* sg	(NEG *every*)
particular or generic nominal collective q.	*some* pl	*all* pl	*no* pl	(NEG *all*)
particular nominal distributive q.	*some* pl	*each* sg	*no* sg	(??NEG *each*)
particular nominal q. over two cases	*one* sg	*both* pl	*neither* sg	(NEG *both*)
particular or generic nominal mass q.	*some* sg	*all* sg	*no* sg	(NEG *all*)
particular or generic adverbial count q.	*partly*	*all*	–	(NEG *all*)
particular or generic adverbial mass q.	*partly*	*all* or *entirely*	–	(NEG *all*) or (NEG *entirely*)
adverbial q. over times, adverbial generic q. over cases	*sometimes*	*always*	*never*	(NEG *always*)
adverbial q. over places	*somewhere*	*everywhere*	*nowhere*	(NEG *everywhere*)
truth conditional connectives	*or*	*and*	*neither ... nor*	(NEG *and*)

Table 1: duality groups of quantifiers

All these cases are obvious instances of existential and universal quantification and their negations. In no group the subnegative is lexicalized. The *partly* groups exhibit a peculiar gap for the negative. The conjunctions *and* and *or* can be subsumed under quantification, as they serve to express that all or some of the conjuncts are true.

The case of negated conjunction needs careful intonation; its status is certainly marginal; one would prefer to say, e.g. *Mary and Paul are not both sick.*

3.2.2 Deontic modality

From the point of view of modal logic, modalities such as possibility and necessity, too, are instances of quantification. Necessity corresponds to truth, or givenness, in all cases out of a given set of alternatives, while possibility means truth in some such cases. The expressions of modality include grammatical forms such as causatives, potentials, imperatives etc. as well as modal verbs, adverbs, adjectives, verbs and nouns. (For a survey of modality, and mood, see Palmer 2001; also article 50 *Verbal mood*, article 58 *Modality*.)

Modal verbs such as those in English and other Germanic languages express various kinds of modality, among them deontic and epistemic. The composition of duality groups out of the same pool of verbs differs for different modalities. Although duality relations constitute basic semantic data for modal verbs, they are hardly taken into account in the literature (e.g. Palmer 2001 or Huddleston & Pullum 2002 do not mention duality relations.).

type of expression	q	dual	negative	subnegative
modal verbs (deontic modality)	*may/can*	*must*	(*must* NEG) (*may* NEG)	(*need* NEG)
adjectives	*possible*	*necessary*	*impossible*	*unnecessary*
German causative deontic verbs	*ermöglichen* ”render possible”	*erzwingen* ”force”	*verhindern* ”prevent”	*erübrigen* ”render unnecessary”
imperative	imperative of permission	imperative of request	(NEG imperative)	–
causative	causative of permission	causative of causation	(NEG causative)	–
verbs of deontic and causal modality	*accept* *allow* *let/admit*	*demand* *request* *let/make/force*	*refuse* *forbid* *prevent*	(*demand* NEG) (*request* NEG) (*force* NEG)

Table 2: Duality groups of deontic expressions

In the groups of modal verbs, *may* can often be replaced by *can*. The second group of modal verbs differs in that they have *shall* as the dual of *may* instead of *must*. Since the meanings of *must* and *shall* are not logically equivalent – they express different variants of deontic modality – *may* and *need* in the two duality groups have different meanings, too, since they are interrelated to *shall* and *must* by logical equivalence relations within their respective duality groups. Thus, the assessment of duality relations may serve as a means of distinguishing meaning variants of the expressions involved.

The vocabulary for the adjective group is rich, comprising several near-synonyms for denoting necessity (*obligatory, mandatory, imperative* etc.) or possibility (*permitted, allowed, admissible* and others). Strictly speaking, each adjective potentially spans a duality group of its own. Again the vocabulary of the subneg type is the most restricted one.

The imperative form has two uses, the prototypical one of request, or command, and a permissive use, corresponding to the first cell of the respective duality group. The negation of the imperative, however, only has the neg-reading of prohibition. The case of permitting not to do something cannot be expressed by a simple imperative and negation. Similarly, grammatical causative forms such as in Japanese tend to have a weak (permissive) reading and a strong reading (of causation), while their negation inevitably expresses prevention, i.e. causing not to. The same holds for English *let* and German *lassen*.

3.2.3 Epistemic modality

In the groups of modal verbs in epistemic use, g-negation of *may* yields a subnegative, unlike the neg-reading of the same form in the deontic group. *Can*, however, yields a neg-reading with negation. Thus, the duality groups exhibit remarkable inconsistencies such as the near-equivalence of *may* and *can* along with a clear difference of their respective g-negations, or the equivalence of the g-negations of non-equivalent *may* and *must*.

type of expression	q	dual	negative	subnegative
modal verbs (1)	*can*	*must*	*can* NEG	*need* NEG
modal verbs (2)	*may*	*must*	*can* NEG	*may* NEG
epistemic adjectives	*possible*	*certain*	*impossible*	*questionable*
adverbs	*possibly*	*certainly*	*in no case*	–
verbs of attitude	*hold possible*	*believe*	*exclude*	*doubt*
adjectives for logical properties	*satisfiable*	*tautological*	*contradictory unsatisfiable*	*(NEG tautological)*
verbs for logical relations	*be compatible with*	*entail*	*exclude*	(NEG *entail*)

Table 3: Duality groups of epistemic expressions

Logical necessity and possibility can be considered a variant of epistemic modality. Here the correspondence to quantification is obvious, as these properties and relations are defined in terms of existential and universal quantification over models (or worlds, or contexts).

3.2.4 Aspectual operators

Aspectual operators deal with transitions in time between opposite phases, or equivalently, with beginning, ending and continuation. Duality groups are defined by verbs such as *begin*, *become* and by focus particles such as *already* and *still*. The German particle *schon* will be analyzed in §5.1 in more detail, as a paradigm case of 'phase quantification'.

The particles of the *nur noch* group have no immediate equivalents in English. See Löbner

(1999: §5.3) for semantic explanations.

type of expression	q	dual	negative	subnegative
verbs of beginning etc.	*begin* *become*	*continue* *stay*	*end* (*become* NEG)	(*NEG begin*) (*NEG stay*)
German aspectual particles with stative operands (1)	*schon* “already”	*noch* “still”	(*noch NEG*) “NEG yet”	(*NEG mehr*) “NEG more”
German aspectual particles with focus on a specification of time	*schon* ”already”	*erst* ”only”	(*noch NEG*) ”NEG yet”	(*NEG erst*) ”NEG still”
German aspectual particles with stative operands (2)	*endlich* “finally”	*noch immer* “STILL”	*noch immer NEG* „STILL NEG“	(*endlich NEG mehr*) “finally NEG more”
German aspectual particles with scalar stative operands (3)	*nur noch*	*noch*	(*NEG nur noch*)	(*NEG mehr*)

Table 4: Duality groups of aspectual expressions

3.2.5 More focus particles, conjunctions

More focus particles such as *only*, *even*, *also* are candidates for duality relations. A duality account of German *nur* (“only”, “just”) is proposed in Löbner (1990: §9). In some of the uses of *only* analyzed there, it functions as the dual of *auch* (“also”). An analysis of *auch*, however, is not offered there. König (1991a, 1991b) proposed to consider causal *because* and concessive *although* duals, due to the intuition that ‘*although p, not q*’ means something like ‘*not (because p, q)*’. However, Iten (2005) argues convincingly against that view.

3.2.6 Scalar adjectives

Löbner (1990: §8) offers a detailed account of scalar adjectives which analyzes them as dual operators on an implicit predication of markedness. The analysis will be briefly sketched in §5.1. According to this view, pairs of antonyms, together with their negations, form duality groups such as [*long*, *short*, (*NEG long*), (*NEG short*)].

Independently of this analysis, certain functions of scalar adjectives exhibit logical relations that are similar to the duality relations. Consider the logical relationships between positive with *enough* and *too* with positive, as well as between equative and comparative:

(24) a. *x is not too short* ≡ *x is long enough*
x is too long ≡ *x is not short enough*

b. *x is not as long as y* ≡ *x is shorter than y*
x is as long as y ≡ *x is not shorter than y*

The negation of *too* with positive is equivalent to the positive of the antonym with *enough*, and the negation of the equative is equivalent to the comparative of the antonym. Antonymy essentially means reversal of the underlying common scale. Thus the equivalences in (24) represent instances of a "duality" relation that is based on negation and scale reversal, another self-inverse operation, instead of being based on negation and subnegation. In view of such data, the notion of duality might be generalized to analogous logical relations based of self-inverse operations.

3.3 Asymmetries within duality groups

The duality groups considered here exhibit remarkable asymmetries. Let us refer to the first element of a duality group as type 1, its dual as type 2, its negative as type 3 and its subnegative as type 4. The first thing to observe is the fact that there are always lexical or grammatical means of directly expressing type 1 and type 2, sometimes type 3 and almost never type 4. This tendency has long been observed for the quantifier groups (see Horn 1972) for an early account, Döhmann (1974a,b) for cross-linguistic data, and Horn (to appear) for a comprehensive recent survey). In addition to the lexical gaps, there is a considerable bias of negation towards type 3, even if the negated operand is type 2. types 3 and 4 are not only less frequently lexicalized; if they are, the respective expressions are often derived from type 1 and 2, if historically, by negative affixes, cf. *n-either*, *n-ever*, *n-or*, *im-possible*. The converse never occurs. Thus, type 4 appears to be heavily marked: on a scale of markedness we obtain 1, 2 < 3 < 4.

A closer comparison of type 1 and type 2 shows that type 2 is marked vs. type 1, too. As for nominal existential quantification, languages are somehow at pain when it comes to an expression of neutral existential quantification. This might at a first glance appear to indicate markedness of existential vs. universal quantification. However, nominal existential quantification can be considered practically "built into" mere predication under the most frequent mode of particular (non-generic) predication: particular predication entails reference, which in turn entails existence. Thus, existential quantification is so unmarked that it is even the default case not in need of overt expression. A second point of preference compared to universal quantification is the degree of elaboration of existential quantification by more specific operators such as numerals and other quantity specifications.

For the modal types, this argumentation does not apply. What distinguishes type 1 from type 2 in these cases, is the fact that irregular negation, i.e. subneg readings of g-negation only occur with type 2 operators; in this respect, epistemic *may* constitutes an exception.

The aspectual groups will be discussed later. So much may, however, be stated. In all languages there are very many verbs that incorporate type 1 'become' or 'begin' as opposed to very few verbs that would incorporate type 2 'continue' or 'stay' or type 3 'stop', and apparently no verbs incorporating 'not begin'.

These observations that result in a scale of markedness of type 1 < type 2 < type 3 < type 4 are tendencies. In individual groups, type 2 may be unmarked vs. type 1. Conjunction is unmarked vs. disjunction, and so is the command use of the imperative opposed to the permission use.

Of course, the tendencies are contingent on which element is chosen as the first of the group. They emerge only if the members of the duality groups are assigned the types they are. Given the perfect internal symmetry of duality groups, the type assignment might seem arbitrary - unless it can be motivated independently. What is needed, therefore, is independent criteria for the assignment of the four types. These will be introduced in §4.2 and §5.3.

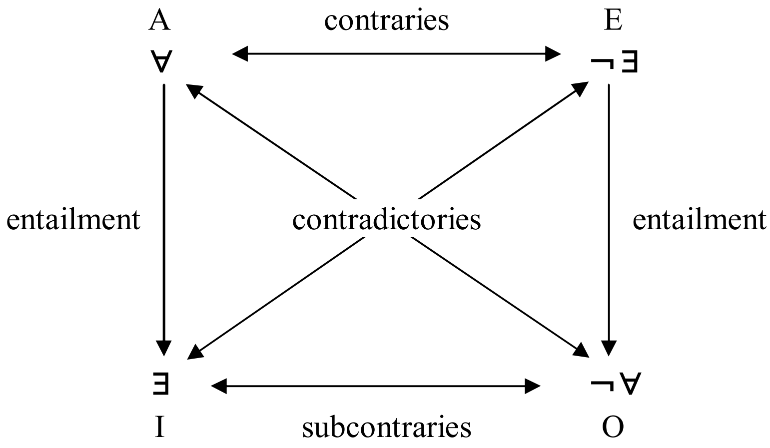

Figure 1: Square of oppositions

4. Semantic aspects of duality groups

4.1 Duality and the Square of Opposition

The quantificational and modal duality groups (Tables 1 , 2, 3) can be arranged in a second type of logical constellation, the ancient Square of Opposition (SqO), established by the logical relations of entailment, contradiction (i.e. neg-opposition), contrariety and subcontrariety. The four relations are essentially entailment relations (cf. Löbner 2002, 2003: §4 for an introduction). They are defined for arbitrary, not necessarily second-order, predicates. The relation of contradictoriness is just neg.

Definition 7: entailment relations for predicate expressions
Let p and p' be arbitrary predicate expressions with the same domain D.
(i) p **entails** p' iff for every a in D, if p is true for a, then p' is true for a.
(ii) p and p' are **contraries** iff p entails Np'.
(iii) p and p' are **subcontraries** iff Np entails p'.

The traditional SqO (Figure 1) has four vertices, A, I, E, O corresponding to $\forall$, $\exists$, $\neg\exists$ and $\neg\forall$, or type 1, 2, 3, 4, respectively, although in a different arrangement than in Figure 1.

The duality square and the SqO depict different, in fact independent logical relations. Unlike the duality square, the SqO is asymmetric in the vertical dimension: the entailment relations are unilateral, and the relation between A and E is different from the one between I and O. The relations in the SqO are basically second-order relations (between first-order predicates), while the duality relations dual and subneg are third-order relations (between second-order predicates).

The entailment relations can be established between arbitrary first-order predicate expressions; the duality relations would then simply be unavailable due to the lack of predicate-type arguments. For example, any pair of contrary first-order predicate expressions such as *frog* and *dog* together with their negations span a SqO with, say, A = *dog*, E = *frog*, I = N *frog*, O = N *dog*. There are also SqQ's of second-order operators which are not duality groups. For example, let A be *more than one* and E *no* with their respective negations O *not more than one/at most one* and I *some/at least one*. The SqO relations obtain, but A and I, i.e. *more than* *one* and *some* are not duals: the dual of *more than one* is *not more than one not*, i.e. *at most one not.* This is clearly not equivalent with *some*.

On the other hand, there are duality groups which do not constitute SqQ's, for example the *schon* group. An SqO arrangement of this group would have *schon* and *noch nicht*, and *noch* and *nicht mehr*, as diagonally opposite vertices. However, in this group duals have different presuppositions, in fact this holds for all horizontal or vertical pairs of vertices. Therefore, since the respective relations of entailment, contrariety and subcontrariety require identical presuppositions, none of these obtains. In Löbner (1990: 210) an artificial example is constructed which shows that the duality relations do not entail the SqO relations even if all four operators share the same domain. There may be universal constraints for natural language which exclude such cases.

4.2 Criteria for the distinction between the four types

4.2.1 Monotonicity: types 1 and 2 vs. types 3 and 4

The most salient difference among the four types is that between types 1 and 2 and types 3 and 4: types 1 and 2 are positive, types 3 and 4 negative. The difference can be capture by the criterion of monotonicity. (Barwise & Cooper 1981) first introduced this property for quantifiers; see also article 43 *Quantifiers.*)

Definition 8: monotonicity

a. An operator q is **upward monotone – mon↑**– if for any operands p and p' if p entails p' then q(p) entails q(p').

b. An operator q is **downward monotone – mon↓**– if for any operands p and p' if p entails p' then q(p') entails q(p).

All type 1 and type 2 operators of the groups listed are mon↑ while the type 3 and type 4 operators are mon↓. Negation inverses entailment: if p entails p' then Np' entails Np. Hence, both N and S inverse the direction of monotonicity. To see this, consider (25):

(25) *have a coke* entails *have a drink* , therefore

a. *every* is mon↑: *every student had a coke* entails *every student had a drink*

b. *no* is mon↓: *no student had a drink* entails *no student had a coke*

Downward monotonicity is a general trait of semantically negative expressions (cf. Löbner 2000: §13). It is generally marked vs. upward monotonicity. Mon↓ operators, including most prominently g-negation itself, license negative polarity items (cf. article 71 *Polarity items*) and are thus specially restricted. Negative utterances in general are heavily marked pragmatically since they require special context conditions (Givón 1975).

4.2.2 Tolerance: types 1 and 4 vs. types 2 and 3

Intuitively, types 1 and 4 are weak as opposed to the strong types 2 and 3. The weak types make weaker claims. For example, one positive or negative case is enough to verify existential or negated universal quantification, respectively, whereas for the verification of universal and negated existential quantification the whole domain of quantification has to be checked. This distinction can be captured by the property of (in)consistency, or (in)tolerance:

Definition 9: tolerance and intolerance
a. An operator q is **tolerant** iff for some neg-pair p and Np of operands, q is true for both p and Np.
b. An operator q is **intolerant** iff it is not tolerant.

Intolerant operators are “strong”, tolerant ones “weak”.

(26) a. intolerant *every*: *every light is off* excludes *every light is on*

b. tolerant *some*: *some lights are on* is compatible with *some lights are of*

An operator q is intolerant iff for all operands p, if q(p) is true then q(Nq) is false, i.e. Nq(Np) is true. Hence an operator is intolerant iff it entails its dual. Unless q is self-dual, it is different from its dual, hence only one of two different duals can entail its dual. Therefore, of two different dual operators one is intolerant and the other tolerant, or both are tolerant. In the quantificational and modal groups, type 2 generally entails type 1, whence type 2 is intolerant and type 1 tolerant. Since negation inverses entailment, type 3 entails type 4 if type 2 entails type 1. Thus in these groups, type 4 is tolerant, and type 3 intolerant. The criterion of (in)tolerance cannot be applied to the aspectual groups in Table 4. This gap will be taken care of in §5.3.

As a result, for those groups that enter the SqO (i.e. the quantificational and modal groups), the four types can be distinguished as follows.

type 1 / I mon↑ tolerant	**type 2 / A** mon↑ intolerant
type 3 / E mon↓ intolerant	**type 4 / O** mon↓ tolerant

Table 5: Type distinctions in duality squares and in the square of opposition

Horn (to appear: 14) directly connects (in)tolerance to the asymmetries of g-negation w.r.t. to types 3 and 4. He states that ‘intolerant q may lexically incorporate S, but tends not to lexicalize N; conversely, tolerant q may lexically incorporate N, but bars lexicalization of S’ (the quotations are modified as to fit the terminology and notation used here). Since intolerant operators are of type 2 and tolerant ones of type 1, both incorporations of S or N lead to type 3.

The logical relations in the SqO can all be derived from the fact that $\forall$ entails $\exists$. They are logically equivalent to $\forall$ being intolerant which, in turn, is equivalent to each of the (in)tolerance values of the other three quantifiers. The monotonicity properties cannot be derived form the SqO relations. They are inherent to the semantics of universal and existential quantification.

4.3 Explanations of the asymmetries

There is considerable discussion as to the reasons for the gaps observed in the SqO groups. Horn (1972) and Horn (to appear) suggest that type 4 is a conversational implicature of type 1 and hence in no need of extra lexicalization: *some* implicates *not all*, since if it were in fact *all*, one would have chosen to say so. This being so, type 4 is not altogether superfluous; some contexts genuinely require the expression of type 4; for example, only *not all*, not *some*, can be used with normal intonation as a refusal of *all*.

Löbner (1990: §5.7) proposes a speculative explanation in terms of possible differences in cognitive cost. The argument is as follows. Assume that the relative unmarkedness of type 1 indicates that type 1 is cognitively basic and that the other types are cognitively implemented as type 1 plus some cognitive equivalents of N, S, or D. If the duality group is built up as [q, Dq, Nq, DNq], this would explain the scale of markedness, if one assumes that application of D is less expensive than application of N, and simultaneous application of both is naturally even more expensive than either. Since we are accustomed to think of D as composed of S and N, this might appear implausible; however, an analysis is offered (see §0 5.2), where, indeed, D is simple (essentially presupposition negation) and S the combined effect of N and D.

Jaspers (2005) discusses the asymmetries within the SqO, in particular the missing lexicalization of type 4 / O, in more breadth and depth than any account before. He, too, takes type 1 as basic, "pivotal" in his terminology, and types 2 and types 3 as derived from type 1 by two different elementary relations, one of them N. The fourth type, he argues, does not exist at all (although it can be expressed compositionally). His explanation is therefore basically congruent with the one in Löbner (1990), although the argument is based not on speculations about cognitive effort, but on a reflection on the character of human logic. For details of a comparison between the two approaches see Jaspers (2005: §2.2.5.2 and §4).

5. Phase quantification

In Löbner (1987, 1989, 1990) a theory of "phase quantification" was developed which was first designed to provide uniform analyses of the various *schon* groups in a way that captures the duality relationships. The theory later turned out to be also applicable to "only" (German *nur*), scalar adjectives and, in fact, universal and existential quantification in a procedural approach. To the extent that the quantificational and modal groups are all derivative of universal and existential quantification, this theory can be considered a candidate for the analysis of all known cases of duality groups, including all cases of quantification. It is for that reason that, somewhat misleadingly, the notion 'phase quantifier' was introduced. The theory will be introduced in a nutshell here. The reader is referred to the publications mentioned for a more elaborate introduction.

Phase quantification is about some first-order predication p; the truth value of p depends on the location of its argument on some scale; for example, p may be true of t only if t is located beyond some critical point on the scale. For a given predicate p and a relevant scale, there are four possible phase quantifications:

(27) (i) p is true of t, but false for some cases lower on the scale

(ii) p is true of t as it is for the cases lower on the scale

(iii) p is false of t as it is for the cases lower on the scale

(iv) p is false of t, but true of some cases lower on the scale.

Alternatively, the four cases can be put in terms of transitions.

(28) (i) up to t on the scale, there is a transition from false to true w.r.t. p

(ii) up to t on the scale, there is no transition from true to false w.r.t. p

(iii) up to t on the scale, there is no transition from false to true w.r.t. p

(iv) up to t on the scale, there is a transition from false to true w.r.t. p

5.1 Instances of phase quantifications

Schon. In the uses of the *schon* group considered here, the particles are associated with the natural focus of a stative sentence p, i.e. of imperfective, perfect or prospective aspect (for the aspectual distinctions see Comrie 1976). Other uses of *schon* and *noch* are discussed in Löbner (1989, 1999). Such sentences predicate over an evaluation time t_e. Consequently, p can be considered a one-place predicate over times. Due to this function of p, type 1, 2, 3, 4 are therefore referred to as *schon*(t_e,*p*), *noch*(t_e,*p*), *noch nicht*(t_e,*p*) and *nicht mehr*(t_e,*p*). The operators are about possible transitions in time between p being true and p being false. *schon*(t_e,*p*) and *noch nicht*(t_e,*p*) share the presupposition that before t_e there was a period of p being false, i.e. Np. *schon*(t_e,*p*) states that this period is over and at t_e, p is true; *noch nicht*(t_e,*p*) negates this: the Np-period is not over, p still is false at t_e. The other pair, *noch*(t_e,*p*) and *nicht mehr*(t_e,*p*) has the presupposition that there was a period of p before. According to *noch*(t_e,*p*) this period at t_e still continues; *nicht mehr*(t_e,*p*) states that it is over, whence p is false at t_e.

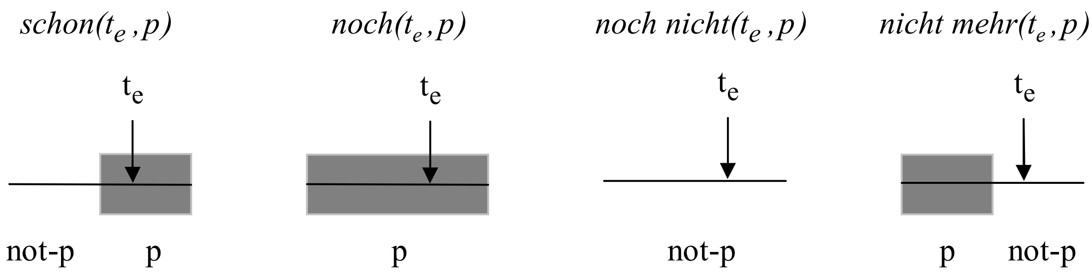

Figure 2: *Phase diagrams for schon*, *noch*, *noch nicht* and *nicht mehr*

operator	relation to *schon*(t_e,*p*)	presupposition: previous state	assertion: state at t_e
schon(t_e,*p*)		not-p	p
noch(t_e,*p*)	dual	p	p
noch nicht(t_e,*p*)	neg	not-p	not-p
nicht mehr(t_e,*p*)	subneg	p	not-p

Table 6: Presuppositions and assertions of the *schon* group

Types 2, 3 and 4 can be directly analyzed as generated from type 1 by application of N and D, where D is just negation of the presupposition.

Mittwoch (1993) and van der Auwera (1993) have questioned the presuppositions of the *schon* group. Their criticism, however, is essentially due to a confusion of types of uses (each use potentially comes with different presuppositions), or of presuppositions and conversational implicatures. Löbner (1999) offers an elaborate discussion, and refutation, of these arguments.

Scalar adjectives. Scalar adjectives frequently come in pairs of logically contrary antonyms such as *long*/*short*, *old*/*young*, *expensive*/*cheap*. They relate to a scale that is based on some ordering. They encode a dimension for their argument such as size, length, age, price, and rank its degree on the respective scale. Pairs of antonyms consist of a positive element +A and a negative element –A (see Bierwisch 1989) for an elaborate general discussion). +A states for its argument that it occupies a high degree on the scale, a degree that is marked against lower degrees. –A states that the degree is low, i.e. marked against higher degrees. The respective negations predicate an unmarked degree on the scale as opposed to marked higher degrees (NEG +A), or marked lower degrees (NEG –A). The criteria of marked degrees are context dependent and need not be discussed here.

Similar to the meanings of the *schon* group, +A can be seen as predicating of an argument t that on the given scale it is placed above a critical point where unmarkedness changes into markedness, and analogously for the other three cases. In the diagrams in Figure 3, unmarkedness is denoted by 0 and markedness by 1, as (un)markedness coincides with the truth value that +A and –A assign to their argument.

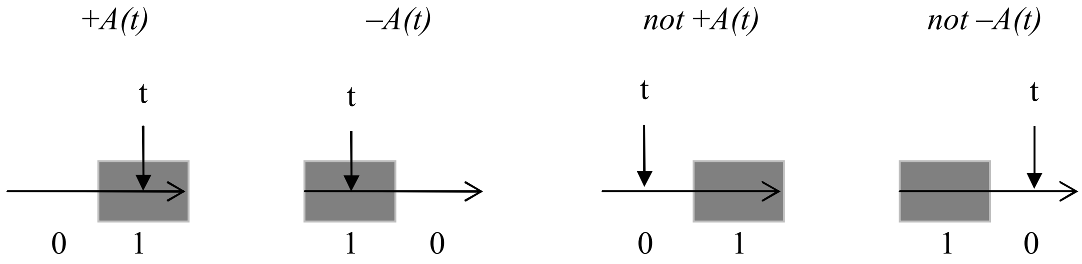

Figure 3: Phase diagrams for scalar adjectives

Other uses of scalar adjectives such as comparative, equative, positive with degree specification, *enough* or *too* can be analyzed analogously (Löbner 1990: §8).

Existential and universal quantification. In order to determine the truth value of quantification restricted to some domain b and about a predicate expressed by p, the elements of b have to be checked in some arbitrary order as to whether p is true or false for them. Existential quantification can be regarded as involving a checking procedure which starts with the outcome 0 and switches to 1 as soon as a positive case of p is encountered in b. Universal quantification would start from the outcome 1 and switch to 0 if a negative case is encountered. This can be roughly depicted as in Figure 4. In the diagrams, b marks the point where the domain of quantification is completely checked. It may be assumed without loss of generality that b is ordered in such a way that there is at most one change of polarity within the enumeration of the total domain.

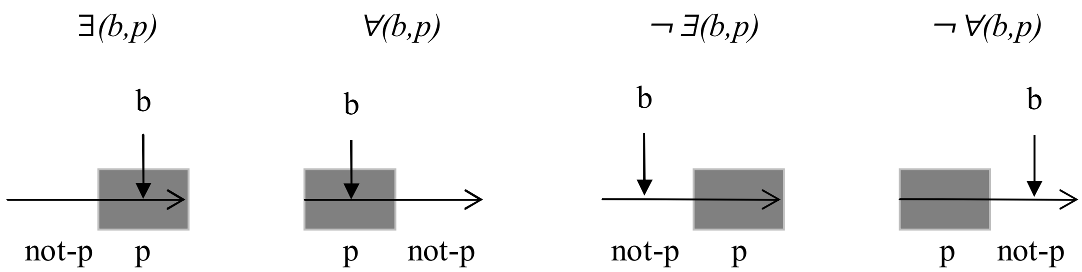

Figure 4: Phase diagrams for logical quantifiers

5.2 The general format of phase quantification

The examples mentioned can all be considered instances of the general format of phase quantification which can be defined as follows. We first need the notion of an 'admissible α-interval'. This is a section of the underlying scale with at most one positive and one negative subsection in terms of p, where α is the truth value of p for the first subsection. An admissible interval may or may not contain a switch of polarity, and this is what phase quantification is all about.

Definition 10: admissible α-intervals in terms of <, p and t
Let p be a predicate expression with domain D, < a partial ordering in D, $t \in D$ and $\alpha = 0$ or 1. The set of admissible α-intervals in terms of <, p and t – AI(α , <, p, t) – is the set of all subsets of D which
(i) are linearly ordered by <
(ii) contain t and some t' < t
(iii) start with a phase of [p] = α
(iv) contain at most one transition from not-p to p below t.

Phase quantification in general is then defined as follows:

Definition 11: phase quantification
Given the conditions of Definition 10,
$PQ(\alpha, <, p, t) \equiv_{df} \exists\forall I\ (I \in AI(\alpha, <, p, t) : p(t))$
For the admissible α-intervals in terms of <, p and t, p is true of t.

With this definition, four phase quantifiers can be defined which form a duality group of the respective four types:

(29) a. $PQ1(<, p, t) \equiv_{df} PQ(0,<, p, t)$

b. $PQ2(<, p, t) \equiv_{df} PQ(1,<, p, t) = D\ PQ1(<, p, t)$

c. $PQ3(<, p, t) \equiv_{df} \neg PQ(0,<, p, t) = N\ PQ1(<, p, t)$

d. $PQ4(<, p, t) \equiv_{df} \neg PQ(1,<, p, t) = N\ D\ PQ1(<, p, t)$

Dual formation D reverses the initial truth value with respect to p, i.e. the truth value of p the admissible interval starts with. D corresponds to presupposition reversal in the case of the *schon* group. Thus the group is built up by N and D, in accordance with the hypothesis mentioned in §4.3. The duality relations can now be formally proved (cf. Löbner 1990: §7). For the details of applying the general format to the *schon* group, scalar adjectives and logical quantifiers, the reader is referred to Löbner (1990: §§7, 8, 10).

5.3 The four types revisited

The four types can be distinguished by the criterion of monotonicity w.r.t. p, or p-monotonicity for short: PQ1 and PQ2 are p-mon↑, and PQ3 and PQ4 are p-mon↓. A second monotonicity criterion, t-monotonicity, concerns the dependence of the truth value of the whole on the position of t. PQ1 and PQ4 are true for t if t is beyond the transition point on the scale, hence PQ1 and PQ4, if true for t, are true for all t' > t within the admissible intervals (t-mon↑) because a further change is impossible. Conversely PQ2 and PQ3, if true for t are true for all t’ < t within those intervals (t-mon↓). For adjectives this means, e.g., that if t is “long” or “not short”, so are all t’ longer than t, while if t is “short” or “not long”, all t’ shorter than t are, too. For quantifiers, the criterion coincides with tolerance (t-mon↑) and intolerance. Similar to tolerance, PQ1 and PQ4, which are true when t is in the second phase, allow for both p and not-p below t. A third monotonicity criterion, s[cale]-monotonicity, concerns the truth value of p at the beginning of the admissible intervals: if α is 0, then p(t) entails p(t’) for all t’ > t (s-mon↑ for PQ1 and PQ3) because t is beyond the transition point; if the intervals start with [p] = 1, p(t) entails p(t’) for all t’ < t (s-mon↓ for PQ2 and PQ4) because any transition point would be above t. S-monotonicity groups together operators with their negatives, i.e. pairs with identical presuppositions.

type	instances			p-monotonicity	t-monotonicity	s-monotonicity
PQ1	*schon*	*long*	*some*	p-mon↑ positive	t-mon↑ (tolerant)	s-mon↑ (neg. presupp.)
PQ2	*noch*	*short*	*all*	p-mon↑ positive	t-mon↓ (intolerant)	s-mon↓ (pos. presupp.)
PQ3	*noch nicht*	*not long*	*no*	p-mon↓ negative	t-mon↓ (intolerant)	s-mon↑ (neg. presupp.)
PQ4	*nicht mehr*	*not short*	*not all*	p-mon↓ negative	t-mon↑ (tolerant)	s-mon↓ (pos. presupp.)

Table 7: Monotonicity properties for phase quantifiers

The three monotonicity properties change with the application of N (p-mon), S (t-mon) and D (s-mon), respectively. PQ1 is basic, ↑ changes to ↓ with application of the respective type-changing operations. The ↓ cases can generally be regarded marked (Löbner 1990: §7). Let us assume that PQ2, PQ3 and PQ4 are derived from PQ1 by N and D as indicated in (29). Then the relevant markedness features are p-monotonicity and s-monotonicity: type 2 is s-mon↓, type 3 p-mon↓, type 4 both s-mon↓ and p-mon↓. If we further assume that p-mon↓ outweighs s-mon↓, we obtain the observed scale of markedness: PQ1 < PQ2 < PQ3 < PQ4.

6. Conclusion

The article offered a general definition of duality as a logical relation between second-order operators. (In the case of dimensional adjectives, their logical type is first-order; their duality rests on an implicit first-order predicate.) The duality relation gives rise to squares of four expressions related in terms of the logical relations of negation, subnegation and duality. Such squares are to be distinguished from the traditional Aristotelian square of oppositions, even if the duality relationships and the Aristotelian oppositions obtain within the same group of four.

It was shown that the known cases of duality can be considered instances of phase quantification, a pattern of second-order predication which deals with transitions of the argument predicate between truth and falsity on some scale the truth-value depends on. This analysis offers a hypothetical explanation for the fact that duality squares are lexicalized in a clearly asymmetric manner.

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