Introduction to Formal Semantics

Handouts

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1 Introduction

1.1 The objectives of formal semantics: Sense relations, truth conditions, inference

(Bach, lecture 1; Carpenter (TLS), chapter 1)

1.1.1 The meaning of "meaning"?

Semantics is the scientific study of the *meaning of signs*. But what is "meaning"?

(1) a. Giving you these flowers means that I love you.
b. Those mountains ahead mean trouble.
c. He said that he would join us, be he didn't mean it.
d. When I say X, I mean Y.
e. *Gatte* means spouse.

(If your native language is not English) How would you translate these examples? Which ones are about semantics?

How about

(2) a. Smoke means fire.
b. *Feuer* means fire.

- Linguistic meaning
  - relates linguistic signs to non-linguistic entities
  - is conventionalized
  - is arbitrary.

1.1.2 What is formal semantics?

Some things formal semantics is **not** concerned with:

- Diachronic semantics:
  1. Why do English *sober* and German *sauber* different things even though they are apparently related?
  2. Why are so many synonyms in English (like *deep* and *profound*)?
- Lexical semantics:
  1. What is the relation between the words *good* and *bad*, *high* and *low* etc., and what is the system behind it?
  2. Is there a common meaning of *in* in the examples *in this room* and *in good mood*?
  3. What is the relation between *chair* as "piece of furniture" and as "leader of a meeting"?
- Stylistics:
  1. What is the difference in meaning between *policeman*, *cop* and *bobby*?

Some things formal semantics **is** concerned with:

- How come that (3) has two totally different meanings, and how can their difference be pinned down?

(3) If a guest comes to the party, he will be surprised

- Does (4a) or (b) entail (c), and why?

(4) a. Nobody managed to prove the theorem.
b. Gauß managed to prove the theorem.
c. There is a proof for the theorem.

- Language is infinite, and meaning is conventionalized, i.e. it must be learned. How do these claims go together?

Formal Semantics is about the meaning of syntactically complex expressions.[1]

[1] This is an historical accident. "Formal semantics" literally means "using formal methods for the study of meaning". Nowadays there is also formal lexical semantics and discourse semantics, but the identification "formal semantics = formal sentence semantics" stuck.

1.1.3 Sense relations

The question is not what meanings are, but how they behave. Formal semantics studies systematic semantic relations between linguistic expressions, such as

Synonymy

- Intuitively:
  - A and B are synonymous iff the have the same meaning
- Vague, sometimes difficult to decide
- Formal approach:
  - A and B are synonymous iff for each sentence S, S is true if and only if $S[B/A]$ is true.

(5) *bachelor* and *unmarried male adult*

Entailment

- Intuitively:
  - A entails B if we can infer from A to B.
- Formal definition:
  - A entails B iff under all conceivable circumstances under which A is true, B must be true as well.

(6) *John owns a bike* entails *John owns something.*

Contradiction

- Intuitively:
  - A and B are contradictory iff they exclude each other.
- Formally:
  - A and B are contradictory iff there are no conceivable circumstances where they could be true at the same time.

(7) *Mary knows every book in the library* and *There is a book in the library that Mary does not know* are contradictory

Inconsistency

- A is inconsistent iff it is contradictory with itself.

(8) This square is round.

Consistency the opposite of inconsistency

(9) Tomorrow is friday.

Tautology

- Intuively: true but contentless
- Formally: always true

(10) a. Every red apple is an apple.
b. There is no largest prime number.

1.1.4 Truth conditions

- "Truth of a sentence" is central notion in definition of sense relations
- Basic methodological principles of formal semantics:
  1. If A and B are sentences, and A is true and B false, then A and B do not have the same meaning. (Cressewell's "Most Certain Principle")
  2. If a person knows the meaning of a sentence, then he or she also know the necessary and sufficient conditions for the truth and falsity of this sentence.
  3. Suppose a person knows the necessary and sufficient conditions for the truth and falsity of a sentence. Then this person knows the meaning of this sentence.

Guiding principles (useful simplifications):

1. **The meaning of a sentence are its truth conditions.**
2. **The meaning of an expression is its contribution to the truth conditions of the sentences it occurs in.**

1.2 Semantics vs. pragmatics I: Speech acts

(Chierchia & McConnell-Ginet, pp 170–203)
Semiotic trichotomy:

- **Syntax:** Study of the *structure* of signs
- **Semantics:** Study of the *meaning* of signs
- **Pragmatics:** Study of the *use* of signs
- Pragmatics builds upon semantics, but cannot be reduced to it.
- "Meaning = truth conditions" suggests that primary purpose of language is descriptive.
- blatantly false
- Consider

(11) a. I hereby christen you Mary-Jane.
b. I promise you to be there at three.
c. I now pronounce you man and wife.

- Under appropriate conditions, these sentences are true **by virtue of their being uttered.**
- Technical term: *performative utterances*
- Same effects can be achieved in a less explicit way

(12) a. Your name be Mary-Jane.
b. I'll be there at three.
c. You are now man and wife.

- Austin (1962): Speech act has three components:
  1. **locution:** uttering a certain sentence of a given language with a given grammatical structure and a given meaning
  2. **illocution:** performing a certain action type (declaring, asking, promising, baptizing, ...)
  3. **perlocution:** Achieving a certain effect by a causal connection between the speech act and a change in the state of the world
- **Performative hypothesis:**
  - All speech acts are explicit performative.

- Usually the performative verb is not pronounced
- For instance, (13a) means the same (in a technical sense) as (b).

(13) a. Are you cold?
b. I hereby ask you whether you are cold.

- Problematic, because the locution of a performative speech act is always true.
- Would render all sentences into tautologies ⇒ unintuitive
- We have to live with the tension between locution (semantics) on the one hand and illocution/perlocution (pragmatics) on the other hand.

2 Basic tools

2.1 Propositional logic

(Gamut I, chapter 2)

- **Principle of Compositionality** ("Frege's principle"):

  The meaning of a complex expression is determined by the meaning of its parts and the way they are combined.

- Meaning of a clause: truth conditions ⇝ determines **truth value**:
  - Bivalent interpretation: Every sentence is either true or false ("tertium non datur"), but not both.
  - Truth values: "True" and "False" ("T" and "F", "1" and "0", "⊤" and "⊥")
- for certain syntactic combinations: **Compositionality of truth values**

(14) a. The power is on and the outside temperature is below freezing point.
b. The power is on.
c. The outside temperature is below freezing point.

(14a) is true iff both (b) and (c) are true, false otherwise.

(15) a. The grapes are too high or you are too short.
b. The grapes are too high.
c. You are too short.

(15a) is true iff at least one of (b) and (c) are true, false otherwise.

(16) a. I do not have the ace of hearts.

b. I have the ace of hearts.

(16a) is true iff (b) is false, and vice versa.

(17) a. If x is a prime number larger than 2, it is an odd number.
b. x is a prime number.
c. x is an odd number.

(17a) is false if (b) is true and (c) false, otherwise it is true.

(18) a. The light is on if and only if the switch is up.
b. The light is on.
c. The switch is up.

(18a) if both (b) and (c) are true or both (b) and (c) are false, otherwise it is false.

- Propositional logic:
  - Simple formal language
  - disregards internal structure of simple clauses
  - conjunction, disjunction, negation, implication and equivalence are only syntactic operations
  - Compositionality of truth values

Definition 1 (Syntax of propositional logic): There is an inifinite number of **propositional variables** $p, q, r, p', p'', q_1, \ldots$.

1. Every propositional variable is a formula.
2. If φ is a formula, then $\neg\varphi$ ("not φ") is a formula.
3. If φ and ψ are formulas, $\varphi \wedge \psi$ ("φ and ψ") is a formula.
4. If φ and ψ are formulas, $\varphi \vee \psi$ ("φ or ψ") is a formula.
5. If φ and ψ are formulas, $\varphi \rightarrow \psi$ ("φ implies ψ") is a formula.
6. If φ and ψ are formulas, $\varphi \leftrightarrow \psi$ ("φ if and only if ψ") is a formula.
7. Nothing else is a formula.

(Note that we use latin letters as variables over atomic clauses and greek letters as meta-variables over complex clauses/formulas.)

Bracketing conventions

- Redundant brackets are optional.
- Connectives are right-associative ($p \to q \to r$ means $(p \to (q \to r))$).
- $\neg$ takes precedence over $\wedge$ takes precedence over $\vee$ takes precedence over $\to$ takes precedence over $\leftrightarrow$ ($\neg p \wedge q \vee r \leftrightarrow s \to t$ means $((((\neg p) \wedge q) \vee r) \leftrightarrow (s \to t))$).

Homework

- Give one own example for each of the sense relations given in 1.1.3 of the handout.
- Chierchia & McConnell-Ginet, chap 4, p. 179, exercise 2
- Gamut I, chap 2, p. 40, exercise 5

References

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Austin, J. L. (1962). *How to do Things with Words*. Harvard University Press, Cambridge (Mass.).

Grice, H. P. (1975). Logic and conversation. In P. Cole and J. Morgan, eds., *Syntax and Semantics 3: Speech Acts*, pp. 41–58. Academic Press, New York.