Right and Wrong Reasons for Compositionality

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In this paper I would like to cast a critical look on the potential reasons for compositionality. I will, in particular, evaluate if and to which extent the most often cited reasons in favor of compositionality, viz. productivity, systematicity and inferentiality – each taken as properties of either language or cognition – may be justly regarded as justifications for compositionality. The results of this investigation will be largely negative: Given reasonable side-constraints, the reason of productivity faces counterexamples of productive languages that cannot be evaluated compositionally. Systematicity has less to do with compositionality than with the existence of semantic categories. The belief that inferentiality is only warranted in compositional languages is a pious hope rather than a certainty. Alternative reasons will be explored at the end of the paper. Before I turn to its reasons, I will explicate the notion of compositionality and say something about its alleged vacuity.

1 The Notion of Compositionality

Although the idea of compositionality is perhaps much older, the locus classicus is Frege’s posthumously published manuscript Logic in Mathematics:

[...] thoughts have parts out of which they are built up. And these parts, these building blocks, correspond to groups of sounds, out of which the sentence expressing the thought is built up, so that the construction of the sentence out of parts of a sentence corresponds to the construction of a thought out of parts of a thought. And as we take a thought to be the sense of a sentence, so we may call a part of a thought the sense of that part of the sentence which corresponds to it. (Frege, 1914/1979, p. 225)

Frege’s claim can be put in more general terms to make it acceptable even to someone who refuses to adopt thoughts and senses into his universe of discourse. To do so, we must distinguish three aspects of Frege’s statement.
First, he claims that there is a part-whole relation between sentences and less complex items of language (‘groups of sounds’). Cases of ambiguous expressions call for a distinction between mereological and syntactic parts (or constituents). The mereological relation is defined as follows:

**Definition 1 (Mereological constituency).** A (spoken or written) utterance $s$ is called a mereological part (or constituent) of an utterance $t$ if and only if for any physical token of $t$ in some region of space in some interval of time, $s$ is physically tokened in the same region and the same interval.

Mereological constituency, thus, is a relation of spatio-temporal co-occurrence. If we, however, contented ourselves with mereological constituency, we would be likely to run into a problem with the second aspect of Frege’s statement: Sentences express thoughts, where for Frege thoughts are nothing but the meanings of sentences. Mereological constituency is a relation apparently too weak to cope with the difficulty that some expressions are either lexically or grammatically ambiguous and need disambiguation in order to be related to their meanings by a function rather than by a many-many relation. The common way to achieve disambiguation is to construe expressions as (abstract) terms combined from syntactic parts and not as (material) utterances combined from mereological parts:

**Definition 2 (Syntactic constituency).** A term $s$ of a language $L$ is called a syntactic part (or constituent) of a term $t$ of $L$ if and only if

1. there is a partial function $\alpha$ from the $n$-th Cartesian product of the set of terms of $L$ into the set of terms of $L$ such that $t$ is a value of the function $\alpha$ with $s$ as one of its arguments and
2. there is a syntactic rule of $L$ according to which $\alpha(s_1,\ldots,s_n)$ is a well-formed term of $L$ if $\alpha$ is defined for $(s_1,\ldots,s_n)$ and if $s_1,\ldots,s_n$ are well-formed terms of $L$.

In short, a term and any of its syntactic parts stand in the relation of value and argument of a syntactic operation. To guarantee unique reference to terms, they should be identifiable by their syntactic parts and the way they have been combined therefrom. This is expressed by the property of unique term identification (cf. Hendriks, 2001):

**Definition 3 (Unique term identification).** The terms of a language are called uniquely identifiable just in case, for any terms $t_1,\ldots,t_n,t'_1,\ldots,t'_n$ and syntactic operations $\sigma,\sigma'$ of the language, the following conditional holds:

$$\sigma(t_1,\ldots,t_n) = \sigma'(t'_1,\ldots,t'_n) \Rightarrow \sigma = \sigma' \land t_1 = t'_1 \land \ldots \land t_n = t'_n.$$
To link terms to utterances, it is common to introduce a surface function for a language, i.e., a surjective function that maps the set of terms onto the set of (types of) utterances.

The third aspect Frege maintains is that the part-whole relation in the linguistic realm, which I will now identify with the relation of syntactic constituency, corresponds to some part-whole relation in the realm of meaning. The sort of correspondence that best fits in place is that of a homomorphism. This analysis of Frege’s statement leads us to the modern (and precise) notion of semantic compositionality as it has been successively developed by Montague (1970/1974), Janssen (1986), Partee, ter Meulen and Wall (1990), and Hodges (2001).

We define the grammar \( G \) of a language \( L \) as a pair
\[
G = \langle T, \Sigma \rangle,
\]
where \( T \) is the set of terms of \( L \) and \( \Sigma \) is the list of basic syntactic operations \( \alpha_1, \ldots, \alpha_j \) of \( L \). The set \( T \) is the closure of a set of primitive terms with regard to recursive application of the syntactic operations. The set of atomic terms is uniquely determined by the grammar as the set of terms that are not in the range of any basic syntactic operation. For technical reasons, we allow terms to have variables \( \xi, \xi_0, \xi_1, \ldots \) as syntactic parts. The set of grammatical terms \( GT(G) \) is a set of terms such that the terms of the set do not contain any variables.

We understand a meaning function \( \mu \) of a language to be a function that maps a subset of the language’s set of grammatical terms to their \( \mu \)-meanings. A grammatical term of the language is called \( \mu \)-meaningful if the term is in the domain of the meaning function \( \mu \). Having introduced all these notions, we can now define the notion of a compositional meaning function:

**Definition 4 (Compositional meaning function).** Let \( \mu \) be a meaning function for a language with grammar \( G \), and suppose that every syntactic part of a \( \mu \)-meaningful term is \( \mu \)-meaningful. Then \( \mu \) is called compositional if and only if, for every syntactic operation \( \alpha \) of \( G \), there is a function \( \mu_\alpha \) such that for every non-atomic \( \mu \)-meaningful term \( \alpha(t_1, \ldots, t_n) \) the following equation holds:
\[
\mu(\alpha(t_1, \ldots, t_n)) = \mu_\alpha(\mu(t_1), \ldots, \mu(t_n)).
\]

A language is called compositional just in case it has a total compositional meaning function. A language, it follows, is compositional just in case the algebra of its grammatical terms
\[
\langle GT(G), \{\alpha_1, \ldots, \alpha_j\} \rangle
\]
is homomorphous to a semantic algebra
\[
\langle \mu[GT(G)], \{\mu_\alpha(1), \ldots, \mu_\alpha(j)\} \rangle.
\]
2 The Alleged Vacuity of Compositionality

The condition of compositionality can fairly easily be trivialized in various ways. Van Benthem was the first to raise this issue:

The general outcome may be stated roughly as ‘anything goes’ – even though adherence to the principle [of compositionality] often makes for elegance and uniformity of presentation. [...] we are entitled to conclude that by itself, compositionality provides no significant constraint upon semantic theory. (van Benthem, 1984, p. 57)

First, for every syntax whatsoever one can take the identity mapping as a compositional meaning function. Every syntax may serve as a semantics for itself. For, in that case we have an isomorphism between syntax and semantics and consequently the homomorphism required by the principle of compositionality is warranted. The price though is a certain form of semantic hyper-distinctness: meanings are as fine-grained as expressions because the meaning function is injective. In languages with a hyper-distinct meaning function there are no synonymous expressions at all. Not even directly logically equivalent sentences would be synonymous. To avoid hyper-distinctness, one may supplement the principle of compositionality by a requirement for non-hyper-distinctness as defined as follows:

Definition 5 (Non-hyper-distinctness). Given a language \( L \) with the set of grammatical terms \( GT(L) \). A meaning function \( \mu \) with domain \( GT(L) \) is called non-hyper-distinct if there are grammatical terms \( s, t \in GT(L) \) such that

\[
s \neq t \\
\mu(s) = \mu(t).
\]

Second, if one does not in some way restrict the surface function, which maps terms to utterances, the syntax algebra one chooses as underlying a language is virtually free. The danger of vacuity with regard to the principle of compositionality of meaning is a side product of the dissolution of ambiguities. After we have disambiguated expressions, our semantic theory, which initially only had to deal with material utterances and meanings, has been amplified by a realm of terms: the syntax algebra. In contrast to the other two realms, the latter figures as a black box in our theory: We can take for granted that the structure and elements of the realm of utterances are directly accessible through observation. Let’s also grant for the moment that we, by intuitive judgements on analyticity, synonymy and other semantic issues, have some, though still meager, access to the structure and elements of the semantics. Terms, though, are nothing but unobservable posits.
To nevertheless explore the syntax of a language, constraints are required that sufficiently strongly reduce the degrees of freedom. Compositionality in itself is too weak a constraint because it only links the realm of terms to the realm of meanings, but leaves the relation between utterances and terms unrestrained.

3 Mereological Surface and Unique Readability

In the attempt to curb the arbitrariness of the surface function, the practice of calling the arguments of syntactic operations the \textit{parts} of the operations’ outcome may easily lead to confusion. For, calling an argument of a syntactic operation, that underlies a certain expression and is responsible for its syntactic structure, a \textit{part} of the expression might suggests that there would be a determinate relation between terms regarded as elements in the syntax and expressions regarded as material utterances. It might arouse the illusion that the formal principle of compositionality in its own light would allow only those terms that stand in a part-whole relation to an utterance to contribute their semantic values to that of the expression. If so, the introduction of \textit{hidden} terms in the syntactic structure of the sentence would be prevented. This sleight of hand lets the formal principle of compositionality appear much stronger than it really is. For, calling a term part of an utterance simply is a category mistake. There is no way to decide whether the term \textit{⌜cook_{tr}⌝} or \textit{⌜cook_{intr}⌝} is a part of the utterance ‘I want my cat to cook’.\footnote{1}

In fact, the principle of compositionality does not exclude the introduction of arbitrary terms in the syntactic analysis of a complex expression for whatsoever reason. It hands out a \textit{carte blanche} to the syntactical analysis of language. Moreover, compositionality downright invites us to postulate hidden terms if other means of achieving compositionality like the postulation of homonymy or the differentiation of syntactic structure seem inappropriate.

The idea of a true mereological part-whole relation between a complex expression and its syntactic parts really makes use of a constraint of syntax that is logically independent from the principle of compositionality. It will here be called the mereological surface property. It is not included in the formal notion of compositionality as it was first presented by Montague (1974), but it had already been anticipated in the Fregean picture as cited above:

\textbf{Definition 6 (Mereological surface property).} Let

\[ f_S : T \rightarrow U \]

\footnote{1I am using inverted commas to denote material utterances, corner quotes for terms and square brackets for concepts.}
Figure 1: The Montagovian picture of semantics. The set of utterances or materially understood expressions $U$ is distinguished from the set of terms $T$. Utterances are the surfaces of terms in the sense that $T$ is surjectively mapped onto $U$ by a surface function $f_S$. Terms are mapped to meanings in the set $M$ by a compositional meaning function $\mu$.

be a surface function for the syntax algebra

$$\mathcal{S} = \langle T; \{\sigma_1, \ldots, \sigma_n\} \rangle$$

and the set of (types of) utterances $U$. Then $\mathcal{S}$ is said to have a mereological surface if and only if it is true that, for every $i = 1, \ldots, n$, if

$$f_S(\sigma_i(t_1, \ldots, t_k, \ldots, t_{ji}))$$

is well defined and occurs in some region of space at some interval of time, then

$$f_S(t_k)$$

also occurs in that region of space at that interval of time, for each $k = 1, \ldots, j_i$.

Figure 1 illustrates the picture of a theory of semantics one attains if one differentiates between terms and utterances. One may justly call it the Montagovian picture because Montague already made this difference and at the same time advocated the principle of compositionality.

The Montagovian picture collapses into the original Frege-picture again, if we require the syntax of a language to be uniquely readable from the utterances of the language:
Definition 7 (Unique readability). Let

\[ f_S : T \to U \]

be a surface function with the set of terms \( T \) and the set of materially individuated utterance (types) \( U \) of the language. Then the syntax

\[ J = \langle T; \sigma_1, ..., \sigma_n \rangle \]

is uniquely readable from \( U \) if and only if \( f_S \) is injective.

The reason for the collapse is obvious: If a syntax algebra

\[ J = \langle T; \{ \sigma_1, ..., \sigma_n \} \rangle \]

with a set of terms as carrier is uniquely readable from a set of utterances \( U \), then the surface function \( f_S \) is bijective – injectivity comes from the definition of unique readability, surjectivity from the definition of a surface function. In this case, the syntax \( J \) is isomorphic to an algebra \( \mathcal{U} \) with the set of utterances as carrier, viz.

\[ \mathcal{U} = \langle U; \{ f_S \circ \sigma_1 \circ f_S^{-1}, ..., f_S \circ \sigma_n \circ f_S^{-1} \} \rangle. \]

The isomorphism makes sure that any homomorphism between the algebra \( J \) of terms and an algebra

\[ \mathcal{M} = \langle M, \{ \mu_1, ..., \mu_n \} \rangle \]

of meanings transfers to the new algebra of utterances. In other words, the compositionality of the syntax \( J \) with respect to the semantic algebra \( \mathcal{M} \) implies that the algebra of utterances \( \mathcal{U} \), in its own right, is compositional with respect to the algebra of meanings. The distinction between terms and utterances becomes superfluous. Mereological structure within utterances becomes syntactical structure. See Figure 2 for an illustration.

Given the numerous lexical and syntactical ambiguities of natural languages, most linguistics would nowadays probably reject the unique readability for natural languages. A more controversially disputed issue, however, is whether natural languages are compositional if the mereological surface property is assumed.

As Lewis (1986), Partee (1984) and Braisby (1998) point out, the question of whether all terms have a mereological surface becomes eminent, e.g., in the case of so-called complex nominals. These include nominal compounds like ‘color television’ and noun phrases with non-predicating adjectives like ‘musical criticism’.

The meaning of ‘color television’ (television showing color), for example, is thought to be not predictable from the meanings of its mereological parts. This
If it is assumed that the syntax is uniquely readable from the utterances of a language, the Montagovian picture of figure 1 collapses into the Fregean picture in which utterances themselves can be compositionally evaluated by a meaning function $\mu^* = \mu \circ f_S^{-1}$.

becomes plausible if one contrasts it with the related examples ‘color palette’ (palette of color), ‘color consultant’ (consultant for color), ‘pocket television’ (television that fits in pockets) and ‘oak television’ (television encased by oak). In all cases the meaning of the complex expression seems to involve some unexpressed – or mereologically surfaceless – relation concept.

Something similar is happening with regard to the adjective ‘musical’ in noun phrases. A musical clock is a clock that produces music; a musical comedy is a comedy that contains music; and musical criticism is criticism of music.

One way to rescue compositionality in these cases is to postulate a relational term in the syntactic structure of the expression that is either surfaceless or whose surface fails to be a mereological part of the complex expression. This would, however, clearly constitute a violation of the mereological surface property. But if one were to give up the requirement of terms to have a mereological surface, how would one then constrain the syntax with regard to the materially individuated utterances?

These considerations show that if one approaches the issue of the compositionality of meaning from an empirical point of view, i.e., in face of concrete linguistic examples, one ends up in a dilemma: Either one tries to avoid van Bentham’s vacuity objection by holding on to the mereological surface prop-
property, then any compositional analysis of compound nouns, certain adjective-noun combinations and many other cases seems defective. Or one drops the requirement of a mereological surface for every term, then compositional analyses of those cases are possible, but also trivial.

4 Asymmetries Between Meaning and Content

The definition of compositionality can easily be applied to mental concepts with contents as their semantic values. However, the problems related to the mereological surface property and to unique readability mark two important asymmetries between debates about the compositionality of meaning (= the semantic value of expressions) and the compositionality of content (= the semantic value of mental concepts).

Regarding the first problem, the debate on whether concepts have to contain their syntactic parts as mereological parts follows considerations completely different from the debate over the mereological surface of terms and has gained wide attention in controversies between classicism and connectionism (Smolensky, 1991/1995; Fodor, 1997). I have discussed those issues elsewhere (Werning, 2003, 2005).

As for the second problem, if we are to validate the principle of compositionality of content in the realm of concepts, we can outright assume that the arguments of our content function are uniquely readable from the structure of concepts and thoughts. For, as Pinker makes very explicit: ‘[...] thoughts, virtually by definition, cannot be ambiguous’ (Pinker, 1997, p. 297). For ambiguity in natural languages roots in the fact that an expression expresses two different concepts or thought. Concepts, however, just are representations of external contents. One cannot represent two different contents by the same concept because a concept must nomologically co-vary with its content in order to have a content.

5 Productivity

The by far most frequently used justification for compositionality in language and cognition is that language and cognition are productive. Fodor (1998) summarizes the productivity argument for compositionality in the following words:

There are infinitely many concepts that a person can entertain. (Mutatis mutandis in the case of natural language: there are infinitely many expressions of L that an L-speaker can understand.) Since people’s representational capacities are surely finite, this infinity of concepts must
itself be finitely representable. In the present case, the demand for finite representation is met if (and as far as anyone knows, only if) all concepts are individuated by their syntax and their contents, and the syntax and contents of each complex concept is finitely reducible to the syntax and contents of its (primitive) constituents. (Fodor, 1998, p. 95)

Fodor then concludes that concepts (mutatis mutandis: expressions) must compose and takes this to be the following claim:

[... ] the claim that concepts compose is the claim that the syntax and the content of a complex concept is normally determined by the syntax and the content of its constituents. (‘Normally’ means something like: with not more than finitely many exceptions. ‘Idiomatic’ concepts are allowed, but they mustn’t be productive.) (Fodor, 1998, p. 94)

Fodor’s caveat regarding idiomatic concepts and, mutatis mutandis, idiomatic expressions has to do with the fact that there, for sure, are idiomatic expression in language and that there maybe are idiomatic concepts in thought. Idiomatic expressions and concepts, however, are typically regarded as exceptions to compositionality. For, their meanings, respectively, contents are commonly regarded not to be a function of the meanings/contents of their syntactic parts. The meaning of ‘red herring’ is not derivable (and hence predictable) from the meanings of ‘red’ and ‘herring’.² A similar violation of compositionality might occur with regard to the concept [red herring] although this is less obvious because it is not clear whether the concepts [red] and [herring] are syntactic constituents of the former.

Now, is Fodor’s argument in the first quote really an argument for the claim in the second quote? In the first quote Fodor states that language and cognition are productive. What Fodor says about productivity might be captured by something like the following definition:

²I do not intend to make any substantial statements about idioms, here, but I should mention that, in an objection to the received view, which is reflected by Nunberg, Sag and Wasow (1994) and according to which some idioms violate semantic compositionality, Westerståhl (1999) argues that idioms can always be embedded in compositional languages. He proposes three ways of doing so: (i) extend the set of atomic expressions by a holophrastic reading of the idiom, (ii) extend the list of syntactic operations so that the literal and the idiomatic reading of the idiom turn out to be outcomes of different syntactic operations, or (iii) take the syntactic parts of the idiom as homonyms of their occurrences in its literal reading and add them to the set of atomic expressions. Westerståhl’s solution, however, strikes me as a little artificial. In our context, though, not much depends on the question whether idioms really are exceptions to compositionality or not.
Definition 8 (Productivity). A language (mutatis mutandis: a conceptual structure) is called productive just in case the following three conditions hold:

a) The syntax of the language (of the conceptual structure) comprises no more than finitely many primitive terms (concepts).

b) The syntax of the language (of the conceptual structure) contains syntactic operations such that potentially infinitely many terms (concepts) can computationally be generated.

c) The meaning (content) function of the language (of the conceptual structure) is computable given the meanings (contents) of the primitive terms (concepts) and the syntax of the language (of the conceptual structure).

Although there are open questions as to whether finite subjects really have the potential to generate infinitely many, or at least potentially infinitely many, expressions or concepts, one might concede that language and cognition indeed are productive in this sense. In my definition the sort of finite reducibility of syntax and semantic value (meaning, respectively, content) Fodor has in mind is accounted for by the computability conditions (b) and (c).

Fodor’s notion of compositionality as stated in the second quote is the same as ours. The question with regard to the validity of Fodor’s argument now is whether the productivity of a language or conceptual structure in the sense of definition 8 implies that the language or conceptual structure is compositional in the sense of definition 4.

The answer is negative: As the following argument shows, languages with a syntactic rule of holophrastic quotation and a non-hyper-distinct meaning function are productive, but not compositional. Productivity does not imply compositionality.

Assume the grammar of a language with the set of expressions $T$ and the meaning function $\mu$ contain the following syntactic operation of quotation:

$$q : T \rightarrow T, \quad s \mapsto \text{‘}s\text{’}$$

such that

$$\mu(q(s)) = s.$$ 

The inclusion of this operation in a language with finitely many primitive expressions warrants that the language is productive because quotation can be iterated and the meaning function is computable. This account of quotation might

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3 Although the formulation might allow for various interpretations, most, if not all compositionality arguments Fodor has given over the time undoubtedly assume a notion of compositionality in the sense of a homomorphism from syntax to semantics.
be called holophrastic quotation because it takes well-formed phrases as unanalyzed wholes and sets them in quotation marks. The meaning of a quotation is the quoted expressions.

It can now be shown that this account of quotation violates compositionality, provided the language to be considered contains synonyms and thus abides by the – as I have argued above – virtually indispensable requirement of non-hyper-distinctness. Assume that the sentences \( "\text{Lou and Lee are brothers}" \) and \( "\text{Lee and Lou are brothers}" \) are synonymous in the language – if you don’t agree that the two sentences are synonymous you can choose any other example of synonymous terms. If we stick to the convention of using corner quotes as our meta-linguistic quotation marks, we can express the synonymy as follows:

\[
\mu ("\text{Lou and Lee are brothers}"
) = \mu ("\text{Lee and Lou are brothers}"
).
\]

(1)

Although the two expressions are synonymous, they are not identical:

\[
"\text{Lou and Lee are brothers}"
\neq
"\text{Lee and Lou are brothers}"
.
\]

(2)

From our definition of the syntactic operation of quotation \( q \) we derive the following:

\[
\mu ("'\text{Lou and Lee are brothers}'"
) = \mu (q("\text{Lou and Lee are brothers}"
)) = "\text{Lou and Lee are brothers}"
.
\]

(3)

\[
\mu ("'\text{Lee and Lou are brothers}'"
) = \mu (q("\text{Lee and Lou are brothers}"
)) = "\text{Lee and Lou are brothers}"
.
\]

(4)

From (2), (3), and (4) we may infer:

\[
\mu (q("\text{Lou and Lee are brothers}"
)) \neq \mu (q("\text{Lee and Lou are brothers}"
))
.
\]

(5)

If we furthermore assume compositionality (see definition 4), there should be a semantic counterpart function \( \mu_q \) for the syntactic operation \( q \) such that:

\[
\mu (q("\text{Lou and Lee are brothers}"
)) = \mu_q (\mu ("\text{Lou and Lee are brothers}"
))
.
\]

(6)

Substitution of identicals according to (1) yields:

\[
\mu (q("\text{Lou and Lee are brothers}"
)) = \mu_q (\mu ("\text{Lee and Lou are brothers}"
))
.
\]

(7)

After another application of compositionality we get:

\[
\mu (q("\text{Lou and Lee are brothers}"
)) = \mu (q("\text{Lee and Lou are brothers}"
))
.
\]

(8)

This contradicts (5). The hypothetical assumption that the language was compositional must be rejected. We have thus given a counterexample to Fodor’s – and not only Fodor’s – supposition that productivity presupposes compositionality. A language with holophrastic quotation is productive, but, non-hyper-distinctness warranted, it is not compositional. Fodor’s argument isn’t valid and productivity is to be rejected as a reason for compositionality.
6 A Compositional Analysis of Quotation

Holophrastic quotation is not the only analysis of quotation in natural language. From the non-compositionality of holophrastic quotation in non-hyper-distinct languages, we can therefore not infer that natural language fails to conform with compositionality because it comprises some syntactic operation of quotation. The inference would only go through if holophrastic quotation were the only possible way to account for quotation in natural language. To show that quotation in natural language can also be analyzed in a compositional way, I will here introduce the method of phonological quotation, which allows us to refer to expressions of natural language by means of a description of their sub-symbolic phonological structure. Unlike holophrastic quotation, phonological quotation can not be conceived of as a function from expressions – taken as unstructured wholes – to their quotations. An earlier, less explicit account of phonological quotation can be found in Davidson (1984), where it is called the spelling theory of quotation.

Let us assume that we be given a productive, compositional and non-hyper-distinct language $L$, which does not yet allow for quotation and which has the following syntax (I will continue to use the same letter for the language and its syntax):

$$L = \langle T, \Sigma \rangle.$$

For reasons of simplicity, I will furthermore assume that the language be uniquely readable (see definition 7) and that the mereological surface property (see definition 6) is satisfied. Under these assumptions we need not distinguish between terms and utterances and may even assume that the surface function be the identity mapping. 4 Let us finally assume that each utterance (and hence each term) be a sequence of primitive phonological parts, i.e., phonemes, or a sequence of any other primitive sub-symbolic parts, e.g., Roman letters or Chinese symbols. The utterance ‘dog’, e.g., is a sequence of the plosive voiced dental consonant ‘d’, the closed-mid back vowel ‘o’ and the plosive voiced velar consonant ‘g’. A sequence is here understood as nothing but a temporally or spatially ordered assembly of matter.

Since the elements of $T$ – due to our assumptions – can be identified with material utterances and since $T$, in a productive language, can be generated from a finite set of atomic terms, we can be certain that there always is a finite set of sub-symbolic mereological parts such that each and every term of the language can be uniquely produced as a sequence thereof.

Having said all this, how can we now proceed to extend the language $L$ by some method of reference to expressions of the language by means of expres-

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4The assumptions are not essential for the argument, but only facilitate the notation.
sions of the language? As we have seen, it would be important if the compositionality of the language did not get lost on the way. The method of phonological quotation promises to accomplish this in that it postulates an additional set of atomic terms \( P \) that comprises names for all sub-symbolic parts necessary to generate each and every term of \( L \). In case of a spoken natural language, this set will comprise names for all phonemes of the language. In case of a written language, a set of names for letters will do the job. We will here assume that \( P \) be a representation of the English alphabet plus the empty space symbol – I am aware that this only amounts to a crude approximation of the elements of phonology:

\[
P = \{ \lceil \text{a} \rceil, \lceil \text{b} \rceil, \lceil \text{c} \rceil, \ldots, \lceil \text{z} \rceil, \lceil \text{ } \rceil \}.
\]

Notice that the symbols \( \text{quote-a-unquote} \), \( \text{quote-b-unquote} \), etc. are supposed to be syntactically primitive. The quotation marks aren’t themselves terms of the language, neither are the letters in between them.\(^5\)

In phonological quotation one construes a means of reference to expressions of the language in that one gives a definite description of the phonological or other sub-symbolic structure of those expressions by syntactic means within the language. In order to do so, we additionally need to introduce a new syntactic operation into \( L \) that, on the level of syntax, reflects the operation of sequencing on the level of semantics. This syntactic operation is the binary operation of concatenation \( \sigma \), where \( \overline{P} \) be the closure of \( P \) with respect to \( \sigma \).

The operation of concatenation

\[
\sigma : \overline{P} \times \overline{P} \to \overline{P}
\]

maps pairs \((X,Y)\) of phonological descriptions of sequences onto phonological descriptions of larger sequences such that the sequence denoted by \( X \) is the first and the sequence denoted by \( Y \) the second (and last) part of the larger sequence. Notice that some, but usually not all sequences so described are terms of the language. The following mappings are examples for the operation of concatenation in the English language with a meaning function \( \mu \) – italics are used to signify sequences in the meta-language and meaning is identified with denotation:

\[
\sigma((\lceil \text{d} \rceil, \lceil \text{o} \rceil)) = \lceil \text{d} \rceil \lceil \text{o} \rceil,
\]

it denotes the sequence \textit{do}. That is,

\[
\mu(\sigma((\lceil \text{d} \rceil, \lceil \text{o} \rceil))) = \mu(\lceil \text{d} \rceil \lceil \text{o} \rceil) = \textit{do}.
\]

\(^5\)The sole exceptions are the first and ninth letter of the alphabet, which correspond to an indefinite article and, respectively, a pronoun in English.
And
\[ \sigma_\leftarrow (\bar{\text{d'} - \text{o' - g'}}) = \bar{\text{d'} - \text{o' - g'}} \]
denoting the sequence *dog*. That is,
\[ \mu(\bar{\text{d'} - \text{o' - g'}}) = \text{dog}, \]
where \( \bar{\text{dog}} \) (consisting of the sequence *dog*) itself is a term of the language
and denotes the set of dogs.

In the algebraic picture, the syntax of the extended language, which is capable
of phonological quotation, now becomes:
\[ L^* = \langle T^*, \Sigma^* \rangle \]
Here, the extended set of terms is
\[ T^* = T \cup P \]
and the extended set of syntactic operations amounts to
\[ \Sigma^* = \Sigma \cup \{ \sigma_\leftarrow \}. \]

What about compositionality in \( L^* \), now? Is the operation of concatenation
semantically compositional? Does it have a semantic counterpart function that
maps the meanings of the arguments of concatenation to the meanings of its
values? – Yes, it does. The function \( \mu_\leftarrow \) just is such a semantic counterpart
function. It maps a pair of sequences onto a larger sequence such that the pair
of sequences make up the first and second part of the larger sequence. Here is
an example:
\[
\mu(\sigma_\leftarrow (\bar{\text{d'} - \text{o' - g'}})) = \mu_\leftarrow (\mu(\bar{\text{d'} - \text{o'}}), \mu(\bar{\text{g'}}))
\]
\[
= \mu_\leftarrow (\text{do}, \text{g})
\]
\[
= \text{dog}. \tag{9}
\]

Equation 9 exemplifies the compositionality condition. Notice that both, the
noun \( \bar{\text{dog}} \) (which is identical to an utterance consisting of the sequence *dog*)
and its phonological description \( \bar{\text{d'} - \text{o' - g'}} \) are terms of \( L^* \).

We may conclude that the existence of quotation in natural language is completely
consistent with the claim that natural language is compositional. Quota-
tion is no exception to compositionality, if only it is analyzed appropriately as
phonological quotation in the sense of this section. The does not infringe the
previous result that languages with holophrastic quotation pose a counterexa-
ple to the implication from productivity to compositionality.
7 Systematicity

If productivity fails to provide a justification for compositionality, what about the often cited justification by the claim that language and cognition are systematic (Fodor & Pylyshyn, 1988). The underlying observation is that intentional and linguistic capacities do not come isolated, but in groups of systematic variants (cf. McLaughlin, 1993). The capacity to imagine a red square in a green circle, e.g., is nomologically correlated with the capacity to imagine a red circle in a green square. Likewise the capacity to understand the sentence ‘The red square is in a green circle’ is nomologically correlated with the capacity to understand the sentence ‘The red circle is in a green square’.

Many authors cite the systematic correlation of linguistic capacities and mental capacities as a reason for semantic compositionality. Minds must have the capacity to compose contents and meanings, so it is argued. Otherwise, they would not show a systematic correlation among intentional and linguistic capacities. If a mind is capable of certain intentional states in a certain intentional mode, it most probably is also capable of other intentional states with related contents in the same mode. Mutatis mutandis: If a mind is capable of understanding certain linguistic expressions, it most probable is also capable of understanding other expressions with related meanings.

Does systematicity really presuppose compositionality? Why isn’t mere syntactic recombination sufficient for systematicity? The systematic correlation among both, contents and meanings, seems, indeed, to imply more than mere syntactic recombination on the level of natural language or conceptual structure. The capacity to think that a child with a red coat is distracted by an old herring is not correlated with the capacity to think that a child with an old coat is distracted by a red herring. The thoughts ought to be correlated, though, if the fact that one is a syntactic re-combination of the other was sufficient for systematic correlation. For, both thoughts are syntactically combined from exactly the same primitives by exactly the same operations. One may, however, well have the capacity to think of red coats and old herrings, even though one lacks the capacity to think of red herrings. That the two thoughts fail to be correlated follows from the fact that the concept [red herring] is idiomatic and thereby violates semantic compositionality.

Likewise, the violation of compositionality by idioms might be held responsible for the fact that the capacity to understand the sentence ‘A child with a red coat is distracted by an old herring’ fails to be systematically correlated with the capacity to understand the sentence ‘A child with an old coat is distracted by a red herring’. From the apparent conditional

\[ \text{violation of compositionality} \Rightarrow \text{violation of systematicity} \]
one may be inclined to infer by contraposition that systematicity presupposes semantic compositionality, both, in the case of cognition and language. This inference may be too quick, though. It has been often overlooked that the phenomenon of systematic correlation is relatively unstable. An often cited pair of systematically correlated sentences (mutatis mutandis: thoughts) is:

(1) Mary loves John.
(2) John loves Mary.

But consider in contrast the following pair:

(3) Mary loves ice cream.
(4) *Ice cream loves Mary.

While (3) is grammatical, (4) is not. This is so despite the fact that the apparent syntactic structure of both sentences is the same: We have a noun phrase followed by verb phrase that takes a direct object. The reason for the violation of grammaticality by (4) seems to be that the verb \( \langle \text{loves} \rangle \) does usually not tolerate an inanimate substance in the subject position.

There are numerous other examples of systematically correlated pairs of sentences that by replacement of some term may be transformed into a pair that is not systematically correlated: Take for example the correlated pair:

(5) The cock pecks the hen.
(6) The hen pecks the cock.

The replacement of the noun \( \langle \text{hen} \rangle \) by the noun \( \langle \text{corn} \rangle \) leads to a pair whose grammaticality is not systematically correlated:

(7) The cock pecks the corn.
(8) *The corn pecks the cock.

Another pair of correlated sentences is:

(9) The boy is reaching for the girl.
(10) The girl is reaching for the boy.

Replacing \( \langle \text{girl} \rangle \) by \( \langle \text{cookie} \rangle \) destroys the correlation:

(11) The boy is reaching for the cookie.
(12) *The cookie is reaching for the boy.
Why is it that in some case a systematic correlation holds, whereas in others it does not? A suitable answer I would suggest is that systematic correlation is warranted only if the permuted words belong to the same Bedeutungskategorie – semantic category or simply category – as defined by Husserl (1970).

Husserl observed that the words and phrases of a language can be organized into classes – the semantic categories – so that (i) for any two expressions of the same class, one expression can replace the other in any non-ambiguous meaningful context without making the context nonsensical and (ii) for any two expressions of different classes the replacement of one expression by the other will make at least some non-ambiguous meaningful contexts nonsensical.\(^6\)

If we apply the notion of a semantic category to the examples (1)–(4), we can say that \(\text{⌜John\⌝}\) and \(\text{⌜ice cream\⌝}\) must belong to different categories. For, a replacement of \(\text{⌜John\⌝}\) by \(\text{⌜ice cream\⌝}\) transforms the meaningful sentence (2) into the meaningless sequence of words (4). \(\text{⌜Mary\⌝}\) and \(\text{⌜John\⌝}\), in contrast, probably belong to the same category. They can be replaced for each other in (1), which leads to (2). (I cannot image a context in which the two proper names cannot be replaced for each other. But this is not certain and it hence remains uncertain if they really belong to the same category.)

Analogous comments can be given to the rest of the above examples. \(\text{⌜cock\⌝}\) and \(\text{⌜hen\⌝}\) probably are in the same category, whereas \(\text{⌜cock\⌝}\) and \(\text{⌜corn\⌝}\) aren’t. Likewise \(\text{⌜boy\⌝}\) and \(\text{⌜girl\⌝}\) belong to the same category, while \(\text{⌜boy\⌝}\) and \(\text{⌜cookie\⌝}\) don’t.

In the above cases taking into account the sameness of category is necessary to decide whether the grammaticality of two sentences systematically correlates or not. Sameness of category, however, also suffices to predict whether a systematic correlation holds or not. For, if two words or phrases occurring in the same sentence belong to the same category, it is always warranted that they can be exchanged for each other without affecting the meaningfulness of the sen-

\(^6\)Husserl takes this classification among expressions to be the consequence of some apriori constitution in the realm of meaning. He postulates a law that

governs the formation of unitary meanings out of syntactic materials falling under definite categories having an a priori place in the realm of meanings, a formation according to syntactic forms which are likewise fixed a priori, and which can be readily seen to constitute a fixed system of forms. (Husserl, 1970, p. 513)

For Husserl the reason why some expressions cannot be replaced for each other in every context without destroying the meaningfulness of the context lies in the fact that the meanings of such expressions belong to different categories. In a remark on Marty he claims that any ‘grammatical division rests on an essential division in the field of meaning’ (Husserl, 1970, p. 500n).
tence. This way a new pair of systematically correlated sentences can always be generated.

Given all this, what role does compositionality play for systematicity? On the one hand, knowledge that a language is compositional is not sufficient to predict systematic correlations. One can’t do without judgements about the sameness of categories. On the other hand, judgements about the sameness of categories themselves suffice to predict systematic correlations. Isn’t this indication enough that systematicity is no good a reason for compositionality?7

8 Inferentiality

For centuries, it had been considered a mystery how any syntactically specified manipulations on internal symbols could preserve semantic properties like truth. It was among others Gödel’s and Turing’s achievement to tell how it goes. What you need is a language (syntax plus semantics) with a logic calculus that is sound, i.e., syntactic derivability must secure semantic validity. If soundness is warranted for the logic of concepts, cognition is possible. Otherwise, it would remain a mystery how internal manipulations of concepts could secure truth-conduction, which is the main goal of cognition. Now, violations of compositionality, at least in some cases, lead to violations of soundness and some authors have alluded to soundness or – as they sometimes call it – inferentiality as a reason for compositionality (Fodor & Pylyshyn, 1988; McLaughlin, 1993).

Assume your logic specify a rule of inference that may be called adjective elimination:

\[
ThisNOUN_1 is aADJNOUN_2
\quad\therefore\quad ThisNOUN_1 isaNOUN_2.
\]

In accordance with the rule of adjective elimination one may syntactically derive

\[
This\ fruit \ is \ a \ pear
\]

from

\[
This\ fruit \ is \ a \ red \ pear.
\]

This derivation is semantically valid: The truth of the premise guarantees the truth of the conclusion. However, if we choose a syntactic \[ADJ NOUN]\-construction that violates compositionality because it is idiomatic, applications of the rule of adjective elimination will no longer be semantically valid. Take

\footnote{For a more elaborate discussion of categories in the context of systematicity and compositionality see Johnson (2004) and Werning (2004).}
Bush’s speech is a red herring  
∴ Bush’s speech is a herring

This is a syntactic derivation in accordance with the rule of adjective elimination. It, however, fails to be a semantically valid inference: Bush’s speech is not a herring, even if it is a red herring. Since in this case we have derivability without validity, soundness is violated.

The reason for the violation of soundness seems to be that the semantic value of the syntactic [ADJ NOUN]-operation as applied to the pair (⌜red⌝,⌜herring⌝) is not a function of the semantic values of ⌜red⌝ and ⌜herring⌝. But can we generalize? Is the soundness of a language’s logic always undermined if compositionality is violated as in the case of idioms? If we could generalize, we, by contraposition, could conclude that the soundness of a logic presupposes that the language be compositional. Since cognition would remain a mystery unless its conceptual structure were to warrant soundness – we might conclude – any conceptual structure should be compositional.

Notice that the mere presence of idioms does not seem to inflect soundness. Take a propositional calculus, for instance. The derivation

Bush’s speech is a red herring and (but) the Mars will be populated anyway
∴ The Mars will be populated anyway

is obviously semantically valid. Somehow only such rules of inference are critical that break up phrases which have been syntactically combined in a semantically non-compositional way.

But even here we find counterexamples: Take again a language with an operation of holophrastic quotation as it has been defined in section 5. We already know that such a language is not compositional, provided that it is not hyper-distinct. Now assume, furthermore, that our language contain a truth-predicate ⌜is true⌝ and consider the following rule of inference:

‘SENTENCE’ is true  
∴ SENTENCE.

An application of this rule of inference, e.g., is the derivation:

‘Snow is white’ is true  
∴ Snow is white.

First, the rule of inference does break up phrases which have been syntactically combined in a semantically non-compositional way, viz. holophrastic quotations. This example is thus different from the one above, which employs
the propositional calculus and does not break up the critical [ADJ NOUN]-
operation. The individual term \[\text{T} \text{\textquoteleft\text{Snow is white}\text{\textquoteright}}\] – corner quotes again for
quotation marks in the meta-language – contains the sentence \[\text{T} \text{\textquoteleft\text{Snow is white}\text{\textquoteright}}\] as a proper syntactic part. The syntactic operation is \(q\) of section 5.

Second, the derivation also is semantically valid. Due to the definition of the
meaning function with \(\mu(q(s)) = s\), it is impossible for the conclusion to be
false if the premise is true, provided that the truth-predicate is interpreted in a
common way (according to the deflationary theory of truth, for example).

It looks as if soundness or inferentiality are unimpeded even in cases where
a rule of inference is to break up a non-compositional syntactic structure. This
might be not the last word, I am ready to concede. As we know, the coinci-
dence of a holophrastic rule of quotation with the truth-predicate easily leads to
paradoxes (the liar paradox is such a case). This issue certainly deserves further
investigation. But so long, inferentiality does not serve as a better reason for
compositionality than productivity and systematicity do.

9 Compositionality and the Principle of Interchangeability of Synonyms

I don’t want to close this paper without giving, at least, a hint where to look for
a reason for compositionality. Where we even have a proof, is the entailment of
the principle of compositionality in the principle of \textit{interchangeability of syn-
onyms salva significatione}. The latter principle says that the substitution of
synonyms for expressions in any linguistic context leaves unchanged the mean-
ing of the context. The principle can be regarded as the meaning (or intensional)
counterpart of the principle of \textit{extensionality, also called the principle of inter-
changeability of co-extensionals salva veritate}. It claims that the substitution of
co-extensional expressions for each other leaves unchanged the truth value of
the embedding linguistic context. While the principle of extensionality is vio-
lated in intensional contexts – contexts like ‘It is necessary that …’, ‘S believes
that …’ – the principle of interchangeability of synonyms \textit{salva significatione}
even pertains to those cases.

The following theorem, which is due to Hodges (2001), proves the equiv-
ance between the principle of compositionality and the principle of inter-
changeability \textit{salva significatione}. Meaning functions are called substitutional
\textit{salva significatione} if they abide by the principle that the substitution of syn-

\[\text{8There is some discussion on the scope of the principle of interchangeability salva significatione. Kripke (1979) tries to construe some counterexamples. I do however think that the principle can be defended. When doing so, one has to take care, though, not to individuate meanings too finely grained. Otherwise one is in danger of jeopardizing non-hyper-distinctness.}\]
onyms for expressions in any linguistic context leaves unchanged the meaning of the context (we write: \( p \equiv \mu q \) if and only if \( \mu(p) = \mu(q) \)).

**Theorem 1.** Let \( \mu \) be a meaning function for a language with grammar \( G \), and suppose that every syntactic part of a \( \mu \)-meaningful term is \( \mu \)-meaningful. Then the following are equivalent:

a) \( \mu \) is compositional.

b) \( \mu \) is substitutional salva significatione, i.e., if \( s \) is a term and \( p_0, \ldots, p_{n-1}, q_0, \ldots, q_{n-1} \) are grammatical terms such that \( s(p_0, \ldots, p_{n-1}|\xi_0, \ldots, \xi_{n-1}) \) and \( s(q_0, \ldots, q_{n-1}|\xi_0, \ldots, \xi_{n-1}) \) are both \( \mu \)-meaningful and, for all \( m < n \),

\[
p_m \equiv \mu q_m,
\]

then

\[
s(p_0, \ldots, p_{n-1}|\xi_0, \ldots, \xi_{n-1}) \equiv \mu s(q_0, \ldots, q_{n-1}|\xi_0, \ldots, \xi_{n-1}).
\]

**Proof.** (a) \( \Rightarrow \) (b). Assuming (a), we prove (b) by induction on the complexity of \( s \). In case \( n = 0 \), \( s \) is a \( \mu \)-meaningful term and the conclusion \( s \equiv \mu s \) is trivial. We now consider the case where \( s \) is the term \( \alpha(t_0, \ldots, t_{m-1}) \). In this case we get

\[
s(p_0, \ldots, p_{n-1}|\xi_0, \ldots, \xi_{n-1})
\]

by substituting the terms \( p_i \) for \( \xi_i \), with \( 0 \leq i < n \), in all syntactic parts \( t_0, \ldots, t_{m-1} \) of the term \( s \). We analogously proceed with

\[
s(q_0, \ldots, q_{n-1}|\xi_0, \ldots, \xi_{n-1})
\]

and thus have:

\[
s(p_0, \ldots, p_{n-1}|\xi_0, \ldots, \xi_{n-1}) = \alpha(t_0(p_0, \ldots, p_{n-1}|\xi_0, \ldots, \xi_{n-1}), \ldots, t_{m-1}(p_0, \ldots, p_{n-1}|\xi_0, \ldots, \xi_{n-1}))
\]

and

\[
s(q_0, \ldots, q_{n-1}|\xi_0, \ldots, \xi_{n-1}) = \alpha(t_0(q_0, \ldots, q_{n-1}|\xi_0, \ldots, \xi_{n-1}), \ldots, t_{m-1}(q_0, \ldots, q_{n-1}|\xi_0, \ldots, \xi_{n-1}))
\]

Since \( s(p_0, \ldots|\xi_0, \ldots) \) and \( s(q_0, \ldots|\xi_0, \ldots) \) are assumed to be \( \mu \)-meaningful, their syntactic parts \( t_i(p_0, \ldots|\xi_0, \ldots) \) and \( t_i(q_0, \ldots|\xi_0, \ldots) \), respectively, are also \( \mu \)-meaningful. By induction hypotheses we may, therefore, presume that

\[
t_i(p_0, \ldots|\xi_0, \ldots) \equiv \mu t_i(q_0, \ldots|\xi_0, \ldots).
\]

According to (a) the \( \mu \)-meanings of \( s(p_0, \ldots|\xi_0, \ldots) \) and \( s(q_0, \ldots|\xi_0, \ldots) \), respectively, are a function of the meanings of their syntactic parts. Thus, the identity of the \( \mu \)-meanings of the parts of both terms implies the identity of the \( \mu \)-meanings of both terms.
Reasons for Compositionality

(b) ⇒ (a). (a) follows at once from the special case of (b) where \( s \) has the form \( \alpha(\xi_0, \ldots, \xi_{n-1}) \). For, in that case (b) just claims the functionality of the relation \( \mu \alpha = \{ (\mu(\xi_0), \ldots, \mu(\xi_{n-1})), \mu(\alpha(\xi_0, \ldots, \xi_{n-1})) | \xi_0, \ldots, \xi_{n-1} \in GT(G) \} \). □

10 Conclusion

We’ve seen that the justification of a principle like compositionality that is central to the semantic analysis of language and to any theory of cognition is everything but an easy task. The first obstacle was to avoid vacuity. In addition to compositionality, the postulation of two further constraints on semantics, non-hyper-distinctness and the mereological surface property, was required. Both constraints have severe side effects, though. The mereological surface property hinders the compositional analysis of language in the light of empirical data massively. For, it forbids the introduction of hidden terms. The requirement of non-hyper-distinctness is responsible for the fact that productivity must no longer be regarded as a reason for compositionality. There are productive languages, that turn out to be non-compositional if hyper-distinctness is not an option. Systematicity, too, fails to provide a justification for compositionality. For, systematicity is a matter solely of membership in semantic categories. Inferentiality falls short of being a reason for compositionality because the soundness of a calculus does apparently not presuppose that the syntactic combinations it breaks up be semantically compositional. The only reason we found was the principle of interchangeability of synonyms. It is logically equivalent to compositionality, but doesn’t this imply that any appeal to it is likely to be a \textit{petitio}? The prospects of a justification of compositionality are not entirely bleak, but less than comfortable.

References


